Helena GASPARS-WIELOCH
Poznan University of Economics and Business,
Faculty of Informatics and Electronic Economy
Department of Operations Research, Poznan
Helena.gaspars@ue.poznan.pl

A SCENARIO-BASED SHORTEST PATH ALGORITHM FOR OPTIMIZING THE SEQUENCE OF CHOICES UNDER UNCERTAINTY

Abstract. The paper presents a procedure based on the shortest path problem (SPP) and on scenario planning. The goal of the method is to find the optimal (with respect to a chosen criterion) sequence of choices under uncertainty, i.e. when at least one parameter of the decision problem is not deterministic. In contrast to existing approaches concerning SPP with uncertainty, we assume that the probability of the occurrence of particular events is not known. The decision rule can be successfully applied for instance to innovative or innovation projects (for both reactive and proactive management) and takes into account the decision maker's attitude towards risk.

Keywords: shortest path problem, sequence of choices, innovative and innovation projects, uncertainty, decision maker, attitude towards risk, optimization model

SCENARIUSZOWY ALGORYTM NAJKRÓTSZEJ ŚCIEŻKI DO OPTYMALIZACJI SEKWENCJI DECYZJI W WARUNKACH NIEPEWNOŚĆI

Streszczenie. Artykuł przedstawia procedurę opartą o zagadnienie najkrótszej ścieżki w grafie (ang. SPP – shortest path problem) i o planowanie scenariuszowe. Celem metody jest znalezienie optymalnej (ze względu na wybrane kryterium) sekwencji decyzji w warunkach niepewności, tj. wówczas, gdy przynajmniej jeden parametr problemu decyzyjnego nie jest deterministyczny. W przeciwieństwie do istniejących podejść dotyczących SPP w warunkach niepewności, przyjmujemy, iż prawdopodobieństwo wystąpienia poszczególnych scenariuszy nie jest znane. Opracowana reguła decyzyjna może z powodzeniem znaleźć zastosowanie przy realizacji projektów innowacyjnych (w przypadku zarządzania zarówno reaktywnego, jak i proaktywnego). Uwzględnia ona nastawienie decydenta do ryzyka.

Słowa kluczowe: zagadnienie najkrótszej drogi w grafie, sekwencja decyzji, projekty innowacyjne, niepewność, decydent, nastawienie do ryzyka, model optymalizacyjny

1. Introduction

The shortest path problem (SPP) is applied to many diverse domains, such us finding directions between physical locations, networking, telecommunications (message routing in communication systems), plant and facility layout, robotics (equipment replacement), transportation (traffic flow through congested cities), project scheduling, cash flow management [1]. In this paper the use of SPP in optimizing the sequence of choices is discussed. We assume that the decision maker (DM) has to make his/her decision under uncertainty. "Uncertainty" and "risk" are interpreted in different ways in the literature [3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20, 23, 24, 27, 38, 41, 42]. Here, risk is related to the possibility that some bad or other than predicted circumstances will happen, while uncertainty involves all situations with non-deterministic parameters – hence, decisions may lead to different consequences and the probability of scenarios is known or not. In the latter case, some probability-like quantities can be often estimated and applied. Optimizing the sequence of choices in an uncertain environment constitutes an extremely vital issue in new product development projects, process development projects, technology-implementation projects, research projects or pharmaceutical development projects since in these cases the deterministic estimation of parameters describing a given decision problem is impossible and historical data connected with similar (or identical) projects do not exist.

The sequence of choices can be represented with the aid of a graph where vertices describe goal states and edges describe transitions. In the contribution, the aforementioned uncertainty concerns parameters characterizing edges. They can be related to profits, expenditures, durations etc. The uncertain shortest path problem has been already investigated in the literature – researchers usually refer to random variables, uncertain variables with regular distribution, fuzzy numbers, probabilities, and interval data [2, 4, 5, 6, 16, 17, 18, 21, 22, 26, 30, 31, 32, 33, 36, 43, 44]. Here, we combine scenario planning [35, 39] with the coefficients of optimism/pessimism being a simple measure of the decision maker's attitude towards risk. Thanks to such a combination the suggested procedure described in the paper allows considering different types of decision makers (risk-neutral, risk-seeking, risk-aversion).

The paper is organized as follows. Section 2 briefly presents the shortest path problem and explains how the SPP can be applied to optimize sequences of choices in an uncertain environment. Section 3 describes a novel scenario-based SPP algorithm that may support the uncertain decision making process. Section 4 provides an illustrative example. Conclusions are gathered in the last part.

2. The shortest path problem and its application to optimizing the sequence of choices under uncertainty

The shortest path problem is a fundamental problem in network optimization and combinatorial optimization [26]. The SPP consists in finding a path between two nodes (vertices) in a undirected, directed or mixed graph (e.g. two intersections on a road map). The sum of the weights of its constituent edges has to be minimized (e.g. path that has the minimum length, path that takes least time to traverse, path that has the maximum reliability) [1]. In the paper, we focus on directed graphs, i.e. graphs containing directed edges (directed arcs). The directed network can be defined by $G=(N,A)$ where N denotes the set of n nodes and A signifies the set of m directed arcs. Each arc $(i,j) \in A$ has an associated cost (or length) c_{ij}. Usually, one of the two nodes of the wanted path is a specified source node s and the second one is a specified sink node t.

The most important procedures for solving the deterministic version of SPP are (depending on the network structure, weights values and the optimization goal): Dijkstra's algorithm, Bellman-Ford algorithm, Bellman-Moore-d'Esopo-Papego algorithm, A* search algorithm, Floyd-Warshall algorithms and Johnson's algorithm. The problem can also be solved by means of the primal or dual optimization model, see e.g. [1]. Note that the constraints of the model and the interpretation of particular variables strictly depend on the DM's target which usually consists in: 1) finding the shortest path from the source node to the sink node or 2) finding the shortest paths from the source node to all other nodes.

As it was mentioned in the introduction, the SPP can be applied, among other things, to the search of optimal sequences of choices. In the case of totally new decisions (concerning for instance innovative or innovation projects[1]), we ought to analyze the stochastic (with known probabilities) or even the strategic (with unknown probabilities) version of SPP. The second suggestion seems more appropriate as the information about previous similar problems (projects), on the basis of which the likelihood could be estimated, is very poor. Thus, both objective and subjective probabilities might be difficult to measure. Additionally, according to [38], the use of probability (understood as frequency) expressed numerically for a single event (such as effects of the execution of an innovative or innovation project) is not suitable since the likelihood should refer to a situation where each event has the opportunity to occur due to its repetitive nature.

When dealing with uncertain issues, scenario planning (SP) may be useful. It is worth stressing that, as a matter of fact, scientists declare different opinions concerning the role of probability in scenario planning. Some of them state that the likelihood should not be applied to SP [28]. Others are convinced that there are many advantages of using probabilities in SP

[1] We assume that innovation projects bring new products and new services, while innovative projects are projects managed on the basis of new methods [37].

[29]. In this contribution the first approach is more reasonable because its author [28] investigates innovation selection problems.

We can define the sequence of new choices under uncertainty with the aid of a graph in the following way. Vertices i describe goal states, directed edges (i,j) describe transitions, weights c_{ij} are deterministic, weights c^k_{ij} are uncertain and given by possible scenario values. The number of scenarios (events) for each arc with a non-deterministic weight may be different. We assume that the set of transitions with scenarios (i.e. non-deterministic transitions) is given by $A(ND)$ and the set of transitions without scenarios (i.e. deterministic transitions) is represented by $A(D)$ where $A = A(D) \cup A(ND)$. The probability distribution for particular sets of events is not known. The aim of the optimization problem is to find the best (the cheapest, the shortest, the most reliable) way to reach a given goal state, especially the final one, starting from state 0 represented by source node s. In connection with the fact that probabilities are not supposed to be applied in this research, we replace them with probability-like quantities. Such an approach is typical of the theory of economics [34]. In order to generate these quantities, we apply the coefficient of optimism/pessimism declared by the DM. Both parameters should satisfy the following conditions: $\alpha, \beta \in [0,1]$ and $\alpha + \beta = 1$, where α denotes the coefficient of pessimism (α is close to 1 for extreme pessimists – risk averse behaviour) and β signifies the coefficient of optimism (β is close to 1 for radical optimists – risk prone behaviour). The level of optimism may be different for particular transitions. Both coefficients are treated as primal data which allow us to generate probability-like quantities, i.e. secondary data. These quantities do not describe a precise probability of occurrence of particular scenarios, but indicate the state of nature "with the biggest subjective chance of occurrence".

3. A scenario-based algorithm for optimizing the sequence of choices under uncertainty

The steps in the novel scenario-based SPP rule are as follows.

1. Present the decision problem on the basis of a directed graph $G=(N,A)$ where N denotes the set of goal states with the source s (represented by all nodes) and A is the set of transitions (represented by all edges). Within A specify $A(ND)$ and $A(D)$.
2. Define parameters concerning particular transitions, e.g. costs c_{ij} for deterministic transitions; costs c^k_{ij} and possible scenarios $s^k_{ij} \in S_{ij}$ for non-deterministic transitions ($s_{ij}{}^k$ signifies scenario k for transition (i,j) and S_{ij} denotes the set of scenarios connected with transition (i,j)).

3. Declare the coefficient of optimism common for the whole decision problem (β) or separate (β_{ij}) for each transition belonging to $A(ND)$.
4. Calculate, for each non-deterministic transition, the value of the measure given by Equation (1) or (2):

$$c(\beta)_{ij} = (1-\beta)\cdot\left(c_{ij}^{\max} - c_{ij}^{\min}\right) + c_{ij}^{\min} \tag{1}$$

$$c(\beta_{ij})_{ij} = (1-\beta_{ij})\cdot\left(c_{ij}^{\max} - c_{ij}^{\min}\right) + c_{ij}^{\min} \tag{2}$$

$$c_{ij}^{\max} = \max_{k}\left\{c_{ij}^{k}\right\} \tag{3}$$

$$c_{ij}^{\min} = \min_{k}\left\{c_{ij}^{k}\right\} \tag{4}$$

For each transition, keep the scenario for which cost c^k_{ij}=$c(\beta)_{ij}$ or c^k_{ij}=$c(\beta_{ij})_{ij}$ and label it by $s(\beta)_{ij}$ (this is the state of nature "with the biggest subjective chance of occurrence"). If for at least one uncertain transition such an event does not exist, go to step 5.
5. For each transition mentioned in the last sentence of the previous step find state of nature s^{min}_{ij} satisfying Equation (5) and set cost c^{min}_{ij} being the cost related to scenario s^{min}_{ij}. This event is also treated as the state of nature "with the biggest subjective chance of occurrence").

$$s_{ij}^{\min} = \arg\min_{s_{ij}^{k}}\left|c(\beta)_{ij} - c_{ij}^{k}\right| \tag{5}$$

6. For each non-deterministic transition, replace $c(\beta)_{ij}$ (or $c(\beta)_{ij}$) and c^{min}_{ij} with c^*_{ij}.
7. Solve the optimization problem (6)-(10) by means of an optimization software package (SAS/OR, MiniZinc, CPLEX, Solver in Excel or "R"). If any changes concerning the graph structure or transition parameters occur before (proactive management) or during (reactive management) the execution of the sequence of choices [19, 25], include those modifications in the model and solve it.

$$\sum_{(i,j)\in A(D)} c_{ij}x_{ij} + \sum_{(i,j)\in A(ND)} c_{ij}^{*}x_{ij} \to \min \tag{6}$$

$$\sum_{(i,j)\in A} x_{ij} = n-1 \qquad i = s = 1 \tag{7}$$

$$\sum_{(j,g)\in A} x_{jg} - \sum_{(i,j)\in A} x_{ij} = -1 \qquad j = 2,\ldots,n-1 \tag{8}$$

$$-\sum_{(i,j)\in A} x_{ij} = -1 \qquad j = t = n \tag{9}$$

$$x_{ij} \ge 0 \qquad i = 1,\ldots,n-1;\ j = 2,\ldots,n \tag{10}$$

where x_{ij} is a non-negative decision variable indicating the number of shortest paths passing through arc (i,j). The number of decision variables is equal to the number of arcs.

In the objective function the sum of all variables multiplied by corresponding costs is minimized. Constraints (except the last one) are equations – their number depends on the number of nodes. s and t still denote the source node and the target node of the graph. Note that parameters related to incoming edges are equal to -1 and parameters connected with outcoming edges are equal to 1.

In order to properly interpret the optimal solution of the model (6)-(10), one needs to establish the course of the shortest path(s) on the basis of all optimal positive values of the decision variables. We recommend using such an optimization computer tool which generates optimal values of primal (x^*_{ij}) and dual (y^*_i) variables. Due to the linear dependency of constraints both in the primal and dual optimization model, optimal values of dual decision variables y^*_i (i.e. variables bounded up with particular nodes) are not expressed explicitly. Hence, if values y^*_i represent the optimal solution of the dual model, values y^*_i+R (where R denotes any constant real number) also constitute the optimal solution of that model. Nevertheless, since the distance between the source node and the first node (i.e. the source) in the network is equal to 0, all optimal values of the dual decision variables should be reduced according to the formula: $y^*_i-y^*_s$, where y^*_s is the original optimal value of the dual decision variable concerning the source node. The final optimal lengths of the shortest paths (from the source node to particular nodes) are equal to the additive inverses of the modified optimal values of the dual decision variables corresponding to consecutive vertices: $d(s,i)=-y^*_i$.

The uncertain model (6)-(10) is based on the original deterministic model presented in [1]. Its purpose is to find the shortest path from the source to any node in the graph and that is why, right-hand parameters in Equations (7)-(9) are equal to n-1, -1 and -1, respectively: we want to find the shortest paths from the source node to all remaining nodes (that is n-1); for i=2,…,n the number of shortest paths after node i to find is 1 less than the number of shortest paths to node i to find.

If the decision maker only desires to set the shortest path from the source node to the target node (without paths reaching other vertices), the model (6)-(10) can be transformed into the following simplified version:

$$\sum_{(i,j)\in A(D)} c_{ij}x_{ij} + \sum_{(i,j)\in A(ND)} c^*_{ij}x_{ij} \to \min \tag{11}$$

$$\sum_{(i,j)\in A} x_{ij} = 1 \qquad i = s = 1 \tag{12}$$

$$\sum_{(j,g)\in A} x_{jg} - \sum_{(i,j)\in A} x_{ij} = 0 \qquad j = 2,..., n-1 \tag{13}$$

$$-\sum_{(i,j)\in A} x_{ij} = -1 \qquad j = t = n \tag{14}$$

$$x_{ij} \geq 0 \qquad i = 1,..., n-1;\ j = 2,..., n \tag{15}$$

4. Illustration

Now, let us illustrate the scenario-based SPP rule. Table 1 contains data about transitions to particular goal states related to an innovative project presented by means of a directed graph $G=(N,A)$ (Figure 1). Each transition requires additional expenditures – some of them are uncertain, the remaining ones are deterministic. In the case of non-deterministic transitions we can notice that the probability of particular scenarios is not given (known) and that the number of states of nature for particular transitions is different. The target of the project manager is to find the cheapest ways to reach all goal states. In order to find these optimal solutions, we are going to apply the novel scenario-based SPP rule. We see that (step 1) set N contains 8 nodes (7 goal states and 1 source), set $A(D)$={(1,2), (2,5), (2,4), (4,5), (3,7), (5,6)}, $A(ND)$={(1,3), (1,4), (3,4), (3,5), (5,7), (6,8), (7,8)}. Parameters concerning each transition, i.e. costs c_{ij} (or c^k_{ij}) for all transitions and possible scenarios $s^k_{ij} \in S_{ij}$ for non-deterministic transitions (step 2), are given in Table 1 (column 2). Now, the project manager should declare his coefficient(s) of optimism (step 3). Let us assume the following values: $\beta_{13}=\beta_{14}=\beta_{34}=\beta_{35}=0.8$, $\beta_{57}=\beta_{68}=0.65$, $\beta_{78}=0.61$. According to Equations (2)-(4) we obtain (step 4): $c(\beta_{13})_{13}=(1-\beta_{13})\cdot(c^{max}_{13}- c^{min}_{13})+c^{min}_{13}=0.2\cdot(21-2)+2=5.8$. All values of the measure $c(\beta_{ij})_{ij}$ are gathered in column 3 of Table 1.

Table 1

Transitions related to an innovative project (example)

Transitions (*i,j*)	**Costs c_{ij} and c^k_{ij} (in thousands of Euros)**	$c(\beta_{ij})_{ij}$
(1,2)	10.0	
(1,3)	s^1_{13}:2.0, s^2_{13}:4.5, s^3_{13}:5.0, s^4_{13}:7.5, s^5_{13}:10.5, s^6_{13}:15.0, s^7_{13}:21.0	5.8
(1,4)	s^1_{14}:22.0, s^2_{14}:14.0, s^3_{14}:25.0, s^4_{14}:7.0, s^5_{14}:10.5	10.6
(2,5)	16.0	
(2,4)	7.5	
(3,4)	s^1_{34}:32.0, s^2_{34}:24.0, s^3_{34}:26.0, s^4_{34}:7.0	12.0
(3,5)	s^1_{35}:9.5, s^2_{35}:14.5, s^3_{35}:15.0, s^4_{35}:27.5, s^5_{35}:15.5, s^6_{35}:21.0	13.1
(4,5)	3.5	
(3,7)	9.5	
(5,6)	10.5	
(5,7)	s^1_{57}:2.0, s^2_{57}:4.0, s^3_{57}:6.0, s^4_{57}:7.5, s^5_{57}:2.5	3.9
(6,8)	s^1_{68}:19.5, s^2_{68}:13.0, s^3_{68}:25.0, s^4_{68}:17.0, s^5_{68}:15.0, s^6_{68}:18.5	17.2
(7,8)	s^1_{78}:15.0, s^2_{78}:9.0, s^3_{78}:16.0, s^4_{78}:27.0, s^5_{78}:11.5	16.0

Source: prepared by the author.

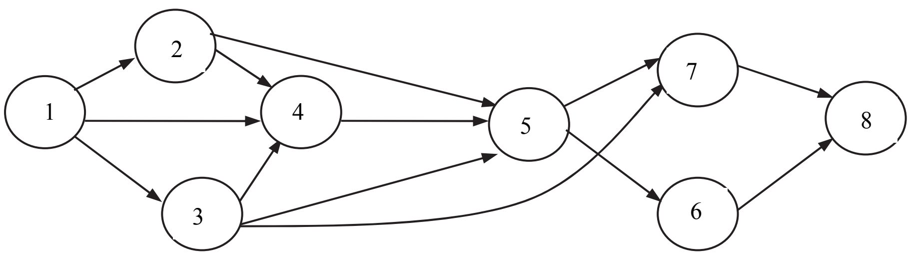

Fig. 1. The directed graph representing the structure of an innovative project.
Source: prepared by the author.

For the last transition (7,8) we can keep scenario $s^3{}_{78}$ which is the only one fulfilling formula (2): $s^3{}_{78}= s(\beta)_{78}$; $c^3{}_{78}= c(\beta_{78})_{78}$=16.0. Other transitions require the use of Equation (5), see step 5: $s^{min}{}_{13}=s^3{}_{13}$, $c^{min}{}_{13}$=5.0; $s^{min}{}_{14}=s^5{}_{14}$, $c^{min}{}_{14}$=10.5; $s^{min}{}_{34}=s^4{}_{34}$, $c^{min}{}_{34}$=7.0; $s^{min}{}_{35}=s^2{}_{35}$, $c^{min}{}_{35}$=14.5; $s^{min}{}_{57}=s^2{}_{57}$, $c^{min}{}_{57}$=4.0; $s^{min}{}_{68}=s^4{}_{68}$, $c^{min}{}_{68}$=17.0. Within step 6 we just replace all costs $c(\beta_{ij})_{ij}$ and $c^{min}{}_{ij}$ with $c^*{}_{ij}$. The optimization model, which is supposed to be solved, contains the following objective function and constraints:

$$\begin{aligned}&(10x_{12} + 16x_{25} + 7.5x_{24} + 3.5x_{45} + 9.5x_{37} + 10.5x_{56}) + \\ &+ (5x_{13} + 10.5x_{14} + 7x_{34} + 14.5x_{35} + 4x_{57} + 17x_{68} + 16x_{78}) \rightarrow \min\end{aligned} \tag{16}$$

$$x_{12} + x_{13} + x_{14} = 7 \tag{17}$$

$$x_{24} + x_{25} - x_{12} = -1 \tag{18}$$

$$x_{34} + x_{35} + x_{37} - x_{13} = -1 \tag{19}$$

$$x_{45} - x_{14} - x_{24} - x_{34} = -1 \tag{20}$$

$$x_{56} + x_{57} - x_{25} - x_{35} - x_{45} = -1 \tag{21}$$

$$x_{68} - x_{56} = -1 \tag{22}$$

$$x_{78} - x_{37} - x_{57} = -1 \tag{23}$$

$$-x_{68} - x_{78} = -1 \tag{24}$$

$$x_{12}, x_{13}, x_{14}, x_{24}, x_{25}, x_{34}, x_{35}, x_{37}, x_{45}, x_{56}, x_{57}, x_{68}, x_{78} \geq 0 \tag{25}$$

The model can be easily solved with the use of such computer tools as SAS/OR, MiniZinc, CPLEX, Solver in MS Excel or "R". The optimal solution of the primal model is as follows: x_{12}=1, x_{13}=3, x_{14}=3, x_{25}=0, x_{24}=0, x_{34}=0, x_{35}=0, x_{45}=2, x_{37}=2, x_{56}=1, x_{57}=0, x_{68}=0, x_{78}=1. The minimal value of the objective function is equal to 109. The interpretation of these results is explained below. Decision variables x_{ij} indicate the number of shortest paths passing through arc (*i*,*j*). Hence, for instance, there is no shortest path passing through arc (3,5), but there are two shortest paths passing through arc (3,7), which means that transition (3,7) provides the opportunity to reach two goal states of the whole project at the lowest cost. Additionally, we can establish the course of the shortest path(s) on the basis of all optimal positive values of the decision variables – the shortest paths pass through arcs (1,2), (1,3), (1,4), (4,5), (3,7), (5,6) and (7,8). The shortest path to goal state 2 (*i*=2) consists of transition (1,2); to goal state 3: (1,3); to goal state 4: (1,4), to goal state 5: (1,4)-(4,5); to goal state 6: (1,4)-(4,5)-(5,6); to goal state 7: (1,3)-(3,7) and to the final goal state: (1,3)-(3,7)-(7,8). The value of the objective function just represents the sum of products of the deterministic or non-deterministic costs multiplied by corresponding primal decision variables being the number of shortest paths – thus, it does not provide any significant information. As it was mentioned in the previous section, the decision variables of the dual model are also vital for the project manager. The dual model does not need to be formulated and solved since many optimization software packages generate automatically reports with the optimal solution of that model. In our example, dual decision variables connected with consecutive constraints (i.e. nodes) of the primal model, have the following optimal values: y^*_1=0, y^*_2=-10, y^*_3=-5, y^*_4=-10.5, y^*_5=-14, y^*_6=-24.5, y^*_7=-14.5, y^*_8=-30.5. We remember that all these values ought to be reduced according to formula: y^*_i-y^*_s. However, in our case y^*_s=y^*_1=0. Therefore, the aforementioned reduction is redundant. The additive inverses related to consecutive vertices equal: $d(s,1)$=-y^*_1=0, $d(s,2)$=-y^*_2=10, $d(s,3)$=5, $d(s,4)$=10.5, $d(s,5)$=14, $d(s,6)$=24.5, $d(s,7)$=14.5, $d(s,8)$=30.5. They represent optimal lengths of shortest paths from the source node to particular nodes. For example, the length of the shortest path to node 7 is equal to 14.5, which means that the minimal cost to reach goal state 7 equals 14.5 thousands of Euros. The shortest path to node i=7 requires the execution of transition (1,3) with a cost amounting to 5 thousands of Euros and (3,7) with a cost equal to 9.5 thousands of Euros (5.0+9.5=14.5). The minimal cost of the whole project equals 30.5 thousands of Euros and can be reached on condition that transitions (1,3), (3,7) and (7,8) are executed.

It is worth emphasizing that the obtained results have a non-deterministic nature and strictly depend on the decision maker's attitude towards risk. For another project manager declaring lower or higher values of the coefficient of optimism for particular non-deterministic costs, parameters in the objective function will be different and this modification will have an impact on optimal values of the decision variables, i.e. on the course and the length of the shortest paths. Note that the sensitivity reports attached to optimal solutions generated by optimization computer tools contain a very useful information concerning the allowable increase and decrease of particular coefficients of the objective functions. In the illustrative example, intervals for these parameters are as follows: $c_{12} \in [3,\infty[$, $c^*_{13} \in [3.5,8.5]$, $c^*_{14} \in [7,12]$, $c_{25} \in [4,\infty[$, $c_{24} \in [0.5,\infty[$, $c^*_{34} \in [5.5,\infty[$, $c^*_{35} \in [9,\infty[$, $c_{45} \in [0,9]$, $c_{37} \in [0,13]$, $c_{56} \in [0,\infty[$, $c^*_{57} \in [0.5,\infty[$, $c^*_{68} \in [6,\infty[$, $c^*_{78} \in [0,27]$. These intervals indicate possible levels of costs for which the optimal solution does not change. Hence, for instance the cost related to transition (1,3) can be equal to 4.5, 5.0 or 7.5 thousands of Euros (see scenarios s^2_{13}, s^3_{13}, s^4_{13}). For all these values the optimal solution is always the same since they belong to interval $c^*_{13} \in [3.5,8.5]$. Of course, we must be aware of the fact that if the optimal value of the decision variable connected with a given transition is positive, the value of the objective function then will be different for each cost. For c^2_{13}=4.5 the objective function is equal to 107.5 and for c^4_{13}=7.5 this function amounts to 116.5 since the decision variable x_{13} (transition (1,3)) has a positive value (it is a basic decision variable). On the other hand, for c^1_{34}=32, c^2_{34}=24, c^3_{34}=26, c^4_{34}=7 (see transition (3,4)), the objective function is always equal to 109 as the decision variable x_{34} has a zero value (it is a non-basic decision variable). Nevertheless, as it was mentioned above, in the primal optimization model for the shortest path problem the objective function has no significant interpretation. The intervals discussed here are very useful for diverse simulation analyses. Thanks to them we are able to predict whether the optimal sequences of choices will change if we consider other levels of the coefficient of optimism (i.e. other states of nature). We see for instance that even if our project manager was more pessimistic about the cost of transition (3,4), the optimal solution would not change since $c^*_{34} \in [5.5, \infty[$. On the other hand, only for β_{14} belonging to [0.71, 1.0] the coefficient (i.e. the cost) connected with x_{14} in the objective function will be from the interval [7,12] – thus, only for radical and moderate optimists the sequences of choices presented above will be optimal.

5. Conclusions

The contribution describes a novel approach referring to the shortest path problem, uncertainty environment and scenario planning. It can be applied, among other things, to optimizing the sequence of choices. The whole decision process is presented by means of a directed graph where edges and nodes show (deterministic and uncertain) transitions and goal states, respectively. The procedure takes into account the decision maker's attitude towards risk and does not require any information about the probability of occurrence of particular states of nature, which is especially advantageous in the case of totally new projects since historical data about previous similar projects do not exist. Instead of the use of the likelihood, consecutive steps of the proposed algorithm lead, separately for all non-deterministic transitions, to the selection of the scenario "with the biggest subjective chance of occurrence". This event is selected on the basis of the coefficient of optimism declared by the decision maker. The procedure gives the possibility to make diverse simulation analyses thanks to the optimization primal model and sensitivity reports generated with the optimal solution. In the future, it would be desirable to investigate the multi-criteria optimization of sequences of choices under uncertainty.

References

1. Ahuja R.K, Magnanti T., Orlin J.B.: Network flows. Theory, algorithms, and applications. Prentice Hall, Upper Saddle River, New Jersey 1993.
2. Deng Y., Chen Y., Zhang Y., Mahadevan S.: Fuzzy Dijkstra algorithm for shortest path problem under uncertain environment. "Applied Soft Computing", Vol. 12, No. 3, p.1231-1237, 2012.
3. Dubois D., Prade H.: Gradualness, uncertainty and bipolarity: making sense of fuzzy sets.

"Fuzzy Sets and Systems", Vol.192, p.3–24, 2012.
4. Feder T., Motwani R., O'Callaghan L., Olston C., Panigrahy R.: Computing shortest path with uncertainty. "Journal of Algorithms", Vol. 62, No. 1, p.1-18, 2007.
5. Frank H.: Shortest paths in probability graphs. "Operations Research", Vol. 17, No. 4, p. 583-599, 1969.
6. Gao Y.: Shortest path problem with uncertain arc lengths. "Computers and Mathematics with Applications", Vol. 62, No. 6, p.2591-2600, 2011.
7. Gaspars-Wieloch H.: Modifications of the Hurwicz's decision rules. "Central European Journal of Operations Research", Vol. 22, No. 4, p. 779–794, 2014.
8. Gaspars-Wieloch H.: A decision rule supported by a forecasting stage based on the decision maker's coefficient of optimism. "Central European Journal of Operations Research", Vol. 23, No. 3, p.579–594, 2015a.
9. Gaspars-Wieloch H.: Modifications of the Omega ratio in decision making under uncertainty. "Croatian Operational Research Review", Vol. 6, No. 1, p.181-194. 2015b.
10. Gaspars-Wieloch H.: Resource allocation under complete uncertainty – case of asymmetric payoffs. "Organization and Management" (Organizacja i Zarzadzanie), Vol. 96, p.247–258, 2016.
11. Gaspars-Wieloch H.: Newsvendor problem under complete uncertainty: a case of innovative products. "Central European Journal of Operations Research", Vol. 25, No. 3, p.561-585, 2017a.
12. Gaspars-Wieloch H.: Innovative projects scheduling with scenario-based decision project graphs. "Contemporary Issues in Business, Management and Education 2017 – Conference Proceedings". 2017b. http://dx.doi.org/cbme.2017.078 .
13. Gaspars-Wieloch H.: A decision rule based on goal programming and one-stage models for uncertain multi-criteria mixed decision making and games against nature. "Croatian Operational Research Review", Vol. 8, No. 1, p.61–75, 2017c.
14. Gaspars-Wieloch H.: Project Net Present Value estimation under uncertainty. "Central European Journal of Operations Research". 2017d. http://dx.doi.org/10.1007/s10100-017-0500-0
15. Gaspars-Wieloch H.: The impact of the structure of the payoff matrix on the final decision made under uncertainty. "Asia-Pacific Journal of Operational Research", Vol. 35, No. 1, 2018. https://doi.org/10.1142/S021759591850001X
16. Hall R.: The fastest path through a network with random time-dependent travel time. "Transportation Science", Vol. 20, No. 3, p.182-188, 1986.
17. Issac P., Campbell A.M.: Shortest path problem with arc failure scenarios. "EURO Journal on Transportation and Logistics", p. 1-25, 2015.
18. Iwamura X.J.K.: New models for shortest path problem with fuzzy arc lengths. "Applied Mathematical Modelling", Vol. 31, No. 2, p.259-269, 2007.

19. Janczura M., Kuchta D.: Proactive and reactive scheduling in practice. “Research Papers of Wroclaw University of Economics”, Vol. 238, p.34-51, 2011.
20. Kaplan S., Barish N.N.: Decision-making allowing for uncertainty of future investment opportunities. “Management Science”, Vol. 13, No. 10, p.569–577, 1967.
21. Karasan O.E., Pinar M.C., Yaman H.: The robust shortest path problem with interval data. Bilkent University, Ankara, Turkey 2001.
22. Klein C.M.: Fuzzy shortest paths. “Fuzzy Sets and Systems”, Vol. 39, No.1, p. 27-41. 1991.
23. Kmietowicz Z.W., Pearman A.D.: Decision theory, linear partial information and statistical dominance. “Omega”, Vol.12, p.391–399. 1984.
24. Knight F. H.: Risk, uncertainty, profit. Hart. Boston MA. Schaffner & Marx. Houghton Mifflin Co. 1921.
25. Kuchta D., Ślusarczyk A.: Application of proactive and reactive project scheduling – case study. “Research Papers of Wroclaw University of Economics”, Vol. 386, p.99-111, 2015.
26. Liu W.: Uncertain programming models for shortest path problem with uncertain arc lengths. “Proceedings of the First International Conference on Uncertainty Theory”. Urumchi, China, Autust 11-19, 2010, pp. 148-153.
27. Merigo J.M.: Decision-making under risk and uncertainty and its application in strategic management. “Journal of Business Economics and Management”, Vol. 2015, No. 1, p. 93–116. 2015.
28. Michnik J.: Scenario planning+MCDA procedure for innovation selection problem. “Foundations of Computing and Decision Sciences”, Vol. 38, No. 3, p. 207-220. 2013.
29. Millet S.M.: Should probabilities be used with scenarios. “Journal of Futures Studies”, Vol. 13, No. 4, p. 61-68. 2009.
30. Mirchandani P.B.: Shortest distance and reliability of probabilistic networks. “Computers and Operations Research”, Vol. 3, No. 4, p. 347-676. 1976.
31. Montemanni R., Gambardella L.M.: The robust shortest path problem with interval data via Benders decomposition. “A Quarterly Journal of Operations Research”, Vol. 3, p. 315-328. 2005.
32. Okada S.: Fuzzy shortest path problems incorporating interactivity among paths. “Fuzzy Sets and Systems”, Vol. 142, No. 3, p. 335-357. 2004.
33. Pascoal M.M.B., Resende M.: The minmax regret robust shortest path problem in a finite multi-scenario model. “Applied Mathematics and Computation”, Vol. 241, p. 88-111. 2014
34. Piasecki K. Intuicyjne zbiory rozmyte jako narzędzie finansów behawioralnych [Intuitive fuzzy sets as a tool f behavioral finances]. Edu-Libri. 2016 (in Polish)
35. Pomerol J.C.: Scenario development and practical decision making under uncertainty. “Decision Support Systems“, Vol. 31, No. 2, p. 197–204. 2001.
36. Sheng Y., Gao Y.: Shortest path problem of uncertain random network. “Computers and Industrial Engineering“, Vol. 99, p. 97-105. 2016.

37. Spalek S.: Innovative vs. Innovation Projects in Organisations, [in:] Wszendybył Skulska E. (ed.): Innowacyjność współczesnych organizacji, TNOiK, Toruń, p. 226-237. 2016.
38. Trzaskalik T.: Wprowadzenie do badan operacyjnych z komputerem [Introduction to operations research with computer]. (2nd ed.). Polskie Wydawnictwo Ekonomiczne, Warsaw 2008. (in Polish)
39. Van der Heijden K.: Scenarios: the art of strategic conversation. John Wiley and Sons, Chichester 1996.
40. Von Mises L.: Human action: a treatise on economics. Yale University Press 1949.
41. von Neumann J., Morgenstern O.: Theory of games and economic behavior. Princeton University Press. Princeton. New York 1944.
42. Ward S., Chapman C.: Transforming project risk management into project uncertainty management. "International Journal of Project Management", Vol. 21, p. 97–105. 2003.
43. Yao J.-S., Lin F.-T.: Fuzzy shortest-path network problems with uncertain edge weights. "Journal of Information Science and Engineering", Vol. 19, p. 329-351. 2003.
44. Zhou J., Yang F., Wang K.: An inverse shortest path problem on an uncertain graph. "Journal of Networks", Vol. 9, No. 9, p. 2353-2359. 2014.