



Jakub Krzysztof GRABSKI¹

¹ *Institute of Applied Mechanics, Poznan University of Technology, Jana Pawła II 24, 60-965 Poznań, Poland*

SOLVING THE BOUNDARY VALUE PROBLEMS BY MESHLESS METHODS FOR REPRESENTING THE SHAPES OF CHILDREN'S HAND BONES OBTAINED FROM RADIOGRAPHS

Keywords: hand bones, Trefftz method, method of fundamental solution, global radial basis function collocation method

The differences between chronological age and bone age can be related to children's endocrine or growing disorders. The radiographs of children's hand bones can be used to assess the bone age. In the literature, one can find many possibilities for describing the shape in computer vision. One of the possibilities to represent the shapes is to solve a boundary value problem described by the Poisson's equation subject to the Dirichlet boundary condition on the given shape, which is a contour of the considered region. This paper proposes meshless methods for representing the children's hand bones shapes obtained from hand radiographs.

Commonly used numerical methods are the finite difference, finite volume, finite element, or boundary element methods. Most of the typical numerical methods need a discretization process of the considering domain at the beginning of the algorithm. An alternative to these mesh-based methods is use of the meshless methods. A typical example of the meshless method is the Trefftz method, which Erich Trefftz proposed in 1926 as an alternative to the Ritz method. In this method, the approximate solution of the considered boundary value problem is assumed as a linear combination of the Trefftz functions, which exactly satisfies the governing equation, not necessarily satisfying the boundary conditions. A kind of the Trefftz method is the method of fundamental solutions, which was proposed in 1964 by Viktor Dmitrievich Kupradze and Merab Aleksandrovich Aleksidze. The role of the Trefftz function in this method is played by the fundamental solution. It is a function of the distance between any point in the considered domain and the source point. These are located outside the considered domain because singularities can occur in these functions at the distance equal to zero. Both in the Trefftz method and method of fundamental solutions, the governing equations are satisfied exactly, and the boundary conditions are satisfied approximately by the collocation method at the finite number of points located on the boundary. The last meshless method considered in this study is the global radial basis function collocation method considered by Edward Kansa in 1990. This method assumes the approximate solution as a linear combination of radial basis functions. The radial basis functions do not satisfy the governing equation and boundary conditions. Thus, they are satisfying approximately by collocation at the finite number of points located inside and on the boundary of the considered region.

In this paper, the solution of the boundary value problem described by the Poisson's equation subject to the Dirichlet boundary condition with the boundary obtained from the hand radiograph is used to represent the children's hand bones. Such an approach is not typical in computer vision and this particular application.