Politechnika Śląska

# Filtry pasywne

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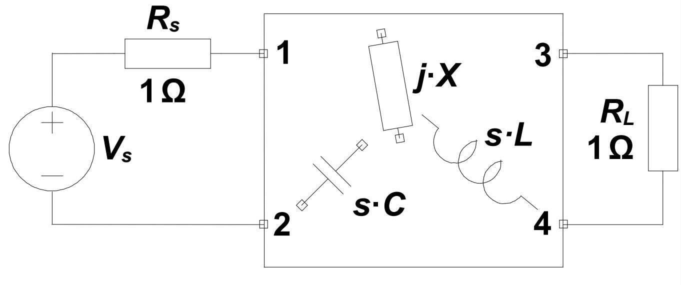
......dla inżyniera elektryka mniej ważne jest rozwiązywanie zadanych równań różniczkowych niż poszukiwanie układów równań różniczkowych (obwodów), których rozwiązanie ma pożądaną właściwość...

Wilhelm Adolf Eduard Cauer
Theorie der linearen Wechselstromschaltungen

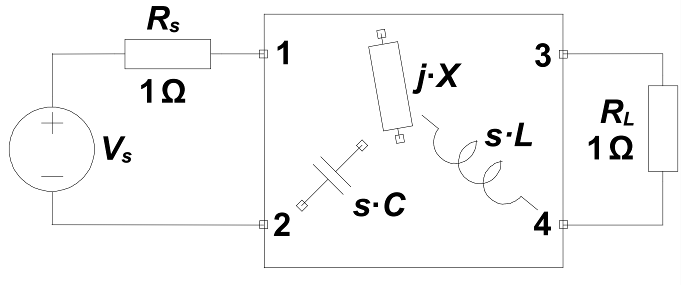
# Plan

- sformułowanie problemu
- filtry pseudoeliptyczne
- od parametrów S do parametrów Y
- filtr prototypowy
- przekształcenia
- wnioski

# Sformułowanie problemu 1

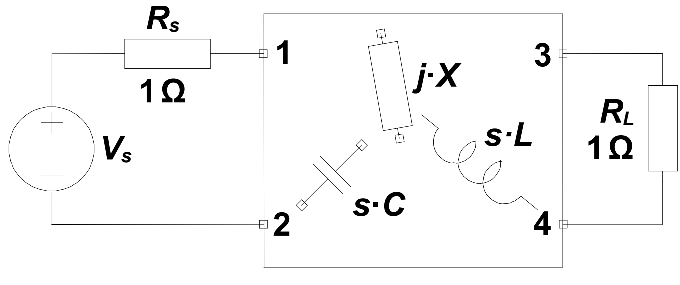


# Sformułowanie problemu 2




$$S_{12}^2(\Omega) = \left|\frac{2V(3,4)}{V_s}\right|^2 = \frac{1}{1+\varepsilon^2 F^2(\Omega)}$$

# Sformułowanie problemu 3

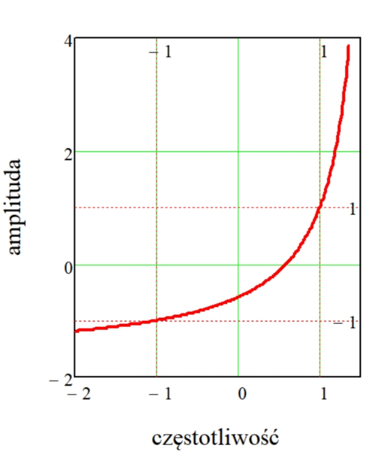



$$S_{12}^2(\Omega) = \frac{1}{1+\varepsilon^2 F^2(\Omega)} \quad \Rightarrow \quad S_{11}^2(\Omega) = \frac{\varepsilon^2 F^2(\Omega)}{1+\varepsilon^2 F^2(\Omega)}$$

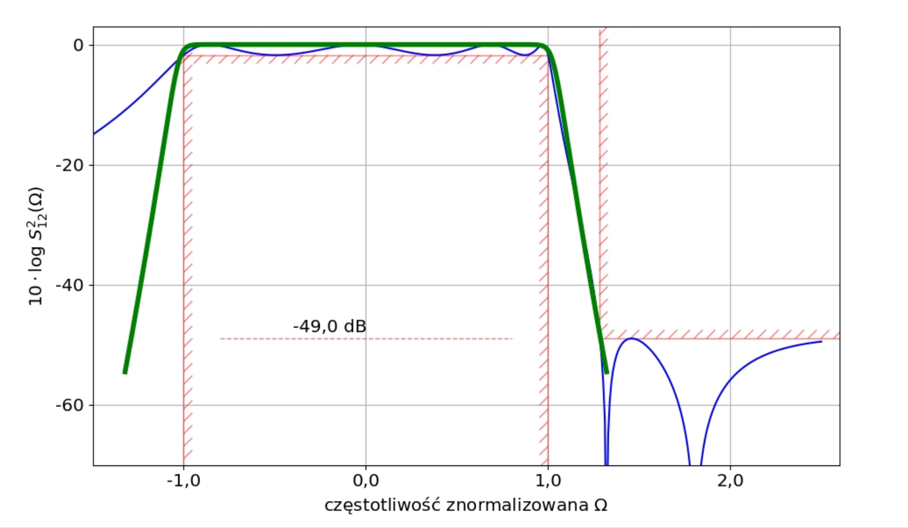
# Funkcja charakterystyczna

$$F(\Omega) = C_N(\Omega) = \cosh\left(\sum_{n=1}^{N} \cosh^{-1}\left(\frac{\Omega - 1/\Omega_n}{1 - \Omega/\Omega_n}\right)\right).$$

$$F(\Omega) = T_N(\Omega) = \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(\Omega)\right).$$



# Filtr pseudoeliptyczny vs. Butterwortha



# Obliczanie $\mathrm{C_N(\Omega)}$

$$\operatorname{cosh}^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right),$$

$$C_N(\Omega) = \operatorname{cosh}\left(\sum_{n=1}^{N} \ln\left(x_n + \sqrt{x_n^2 - 1}\right)\right),$$

$$\text{gdzie:} \quad x_n = (\Omega - 1/\Omega_n)/(1 - \Omega/\Omega_n).$$

$$C_N(\Omega) = \frac{1}{2}\left( \underbrace{\prod_{n=1}^{N} \left(x_n + \sqrt{x_n^2 - 1}\right)}_{W_N(x_n) + P_N(x_n)\sqrt{x_n^2 - 1}} + \prod_{n=1}^{N} \left(x_n - \sqrt{x_n^2 - 1}\right)\right).$$

# Obliczanie $\mathrm{C_N}(\Omega)$ – finał

$$\mathrm{C_N}(\Omega) = \frac{\mathrm{F_N}(\Omega)}{\mathrm{D_N}(\Omega)}$$

$$\mathrm{D}(\Omega) = \prod_{\mathrm{n}=1}^{\mathrm{N}} \Big(1 - \Omega/\Omega_\mathrm{n}\Big).$$

$$\begin{bmatrix} \mathrm{F_n}(\Omega) \\ \mathrm{V_n}(\Omega) \end{bmatrix} = \begin{bmatrix} \Omega - \dfrac{1}{\Omega_\mathrm{n}} & (\Omega^2 - 1)\sqrt{1 - \dfrac{1}{\Omega_\mathrm{n}^2}} \\ \sqrt{1 - \dfrac{1}{\Omega_\mathrm{n}^2}} & \Omega - \dfrac{1}{\Omega_\mathrm{n}} \end{bmatrix} \begin{bmatrix} \mathrm{F_{n-1}}(\Omega) \\ \mathrm{V_{n-1}}(\Omega) \end{bmatrix}.$$

$$\mathrm{F_0}(\Omega) = 1 \quad \mathrm{V_0}(\Omega) = 0.$$

# Wzór rekurencyjny

$$F_n(\Omega) = (\Omega - \frac{1}{\Omega_n})F_{n-1}(\Omega) +$$

$$+ (\Omega - \frac{1}{\Omega_{n-1}})\sqrt{\frac{1 - 1/\Omega_n^2}{1 - 1/\Omega_{n-1}^2}}F_{n-1}(\Omega) -$$

$$- (1 - \frac{\Omega}{\Omega_{n-1}})^2\sqrt{\frac{1 - 1/\Omega_n^2}{1 - 1/\Omega_{n-1}^2}}F_{n-2}(\Omega)$$

# Obliczanie $S_{12}(s)$ i $S_{11}(s)$

$$S_{11}(s) = \frac{Q(s)}{E(s)}, \qquad S_{21}(s) = \frac{P(s)}{\varepsilon E(s)},$$

$$P(s) = (-j)^K \prod_{k=1}^{K} (s - j \cdot \Omega_k). \qquad Q(s) = (-j)^N \prod_{n=1}^{N} (s - j \cdot \overbrace{\Omega'_n}^{F_N(\Omega'_n)=0}),$$

$$\varepsilon = \left| \frac{Q(s)}{P(s)} \right|_{s=\pm j} \frac{1}{\sqrt{10^{0,1 \cdot \mathsf{RL}} - 1}},$$

# Obliczanie $S_{12}(s)$ i $S_{11}(s)$ cd

$$\{e_n : \ \Re(r_n) < 0 \ \ \text{oraz} \ \ P^2(e_k)/\varepsilon^2 + Q^2(e_k) = 0\}$$

$$E(s) = \varepsilon_R \prod_{n=1}^{N} (s - e_n),$$

natomiast:

$$\varepsilon_R = \begin{cases} 1 & \text{gdy} \quad K < N, \\ \dfrac{\varepsilon}{\sqrt{\varepsilon^2 - 1}} & \text{gdy} \quad K = N. \end{cases}$$

# $S \mapsto Y$

$$Y_{in}(s) = \frac{1 - S_{11}(s)}{1 + S_{11}(s)} = \frac{E(s) - Q(s)}{E(s) + Q(s)}.$$

$$\begin{aligned}
n_1(s) &= \mathsf{Re}[\mathsf{Ev}(E - Q)] + j \cdot \mathsf{Im}[\mathsf{Odd}(E - Q)], \\
m_1(s) &= \mathsf{Re}[\mathsf{Odd}(E - Q)] + j \cdot \mathsf{Im}[\mathsf{Ev}(E - Q)], \\
n_2(s) &= \mathsf{Re}[\mathsf{Ev}(E + Q)] + j \cdot \mathsf{Im}[\mathsf{Odd}(E + Q)], \\
m_2(s) &= \mathsf{Re}[\mathsf{Odd}(E + Q)] + j \cdot \mathsf{Im}[\mathsf{Ev}(E + Q)].
\end{aligned}$$

# $S \mapsto Y$ cd

$$y_{11}(s) = \begin{cases} \dfrac{m_2(s)}{n_2(s)} & \text{gdy} \;\; N \;\; \text{jest parzyste,} \\ \dfrac{n_2(s)}{m_2(s)} & \text{gdy} \;\; N \;\; \text{jest nieparzyste,} \end{cases}$$

$$y_{22}(s) = \begin{cases} \dfrac{m_2(s)}{n_2(s)} & \text{gdy} \;\; N \;\; \text{jest parzyste,} \\ \dfrac{n_1(s)}{m_2(s)} & \text{gdy} \;\; N \;\; \text{jest nieparzyste,} \end{cases}$$

$$y_{12}(s) = y_{21}(s) = \begin{cases} \dfrac{j \cdot P(s)}{\varepsilon \cdot n_2(s)} & \text{gdy} \;\; N \;\; \text{jest parzyste,} \\ \dfrac{P(s)}{\varepsilon \cdot m_2(s)} & \text{gdy} \;\; N \;\; \text{jest nieparzyste.} \end{cases}$$

- Wilhelm A.E. Cauer.
  Theorie der linearen Wechselstromschaltungen.
  Akademische Verlagsgesellschaft Becker & Erler Kom.-Ges., Leipzig, 1941.
- J. L. Herrero and G. Willoner.
  Synthesis of Filters.
  Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1966.

# Niezmiennik

$$n_1(s) \cdot n_2(s) - m_1(s) \cdot m_2(s) = P^2(s)/\varepsilon^2.$$

# Niezmiennik cd

$$y_{11}(s) \cdot y_{22}(s) - 1 = y_{21}^2(s).$$

$$r_{11_n} \cdot r_{22_n} = r_{21_n}^2 \quad \text{dla} \quad n = 1, \ldots, N.$$

# Filtry parzystego rzędu – $S_{11}$

$$Y_{in} = \frac{y_{11} + y_{22} \cdot y_{11} - y_{12} \cdot y_{21}}{1 + y_{22}}.$$

$$y_{11}(s) = \frac{m_2(s)}{n_2(s)}, \qquad y_{22}(s) = \frac{m_1(s)}{n_2(s)} = y_{11}(s), \qquad y_{12}(s) = y_{21}(s) = \frac{jP(s)/\varepsilon}{n_2(s)}.$$

$$Y_{in} = \frac{\frac{m_2}{n_2} + \overbrace{\frac{m_1 m_2}{n_2^2} + \frac{P^2/\varepsilon^2}{n_2^2}}^{\uparrow}}{1 + \frac{m_1}{n_2}} = \frac{\frac{m_2}{n_2} + \overbrace{\frac{n_1 n_2}{n_2^2}}^{\downarrow}}{1 + \frac{m_1}{n_2}} = \frac{m_1 + n_1}{n_2 + m_2} = \frac{E - Q}{E + Q}$$

# Filtry parzystego rzędu – $S_{12}$

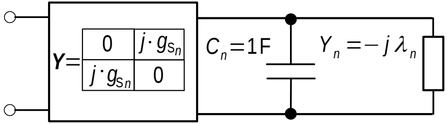
$$K_u = \frac{-\,y_{21}}{(1+y_{22}) + y_{11} + y_{22}\cdot y_{11} - y_{21}\cdot y_{21}}.$$

$$K_u = \frac{-jP/\varepsilon/n_2}{1 + \frac{m_1}{n_2} + \frac{m_2}{n_2} + \underbrace{\frac{m_1 m_2}{n_2^2} + \frac{P^2/\varepsilon^2}{n_2^2}}_{\to}} =$$

$$= \frac{-jP/\varepsilon/n_2}{1 + \frac{m_1}{n_2} + \frac{m_2}{n_2} + \underbrace{\frac{n_1 n_2}{n_2^2}}_{\to}} = \frac{-jP/\varepsilon}{n_1 + m_1 + n_2 + m_2} = \frac{-jP/\varepsilon}{2E}.$$

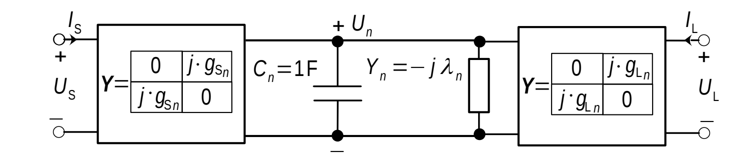
# Obwód prototypowy – wejście

$$y_{11}(s) = r_{11_0} + \sum_{n=1}^{N} \frac{r_{11_n}}{s - j \cdot \lambda_n}, \quad \text{gdzie} \quad r_{11_n} > 0 \quad \text{dla} \quad n = 1, \dots, N.$$




$$g_{S_n} = \sqrt{r_{11_n}}.$$

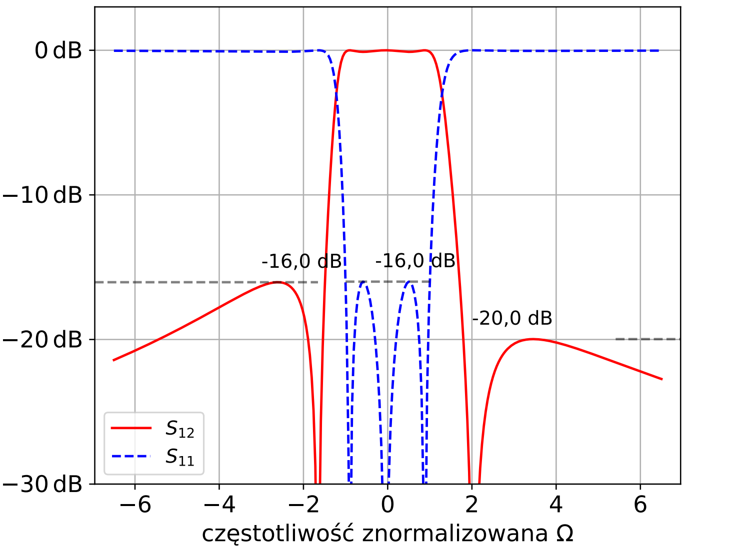
# Obwód prototypowy – we. i wy.




$$g_{S_n} = \sqrt{r_{11_n}}, \qquad g_{L_n} = \frac{r_{21_n}}{\sqrt{r_{11_n}}} = \sqrt{r_{22_n}}.$$

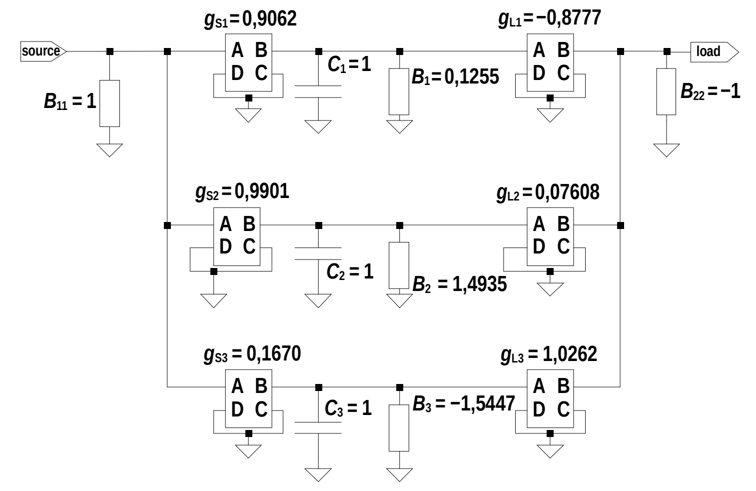
$$y_{11_n} = \frac{r_{11_n}}{s - j \cdot \lambda_n}, \qquad y_{22_n} = \frac{r_{22_n}}{s - j \cdot \lambda_n}, \qquad y_{21_n} = y_{12_n} = \frac{r_{21_n}}{s - j \cdot \lambda_n}.$$

# Przykład 1/5



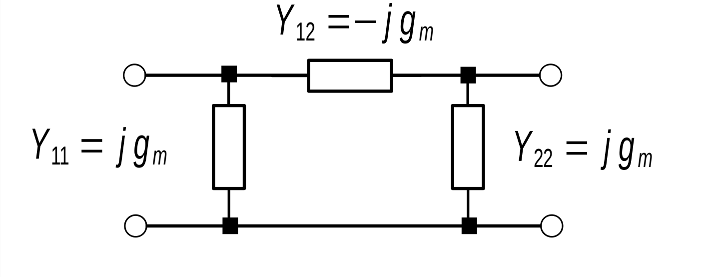
0 dB
−10 dB
−20 dB
−30 dB
-16,0 dB
-16,0 dB
-20,0 dB
$S_{12}$
$S_{11}$
−6
−4
−2
0
2
4
6
częstotliwość znormalizowana Ω

# Przykład 2/5



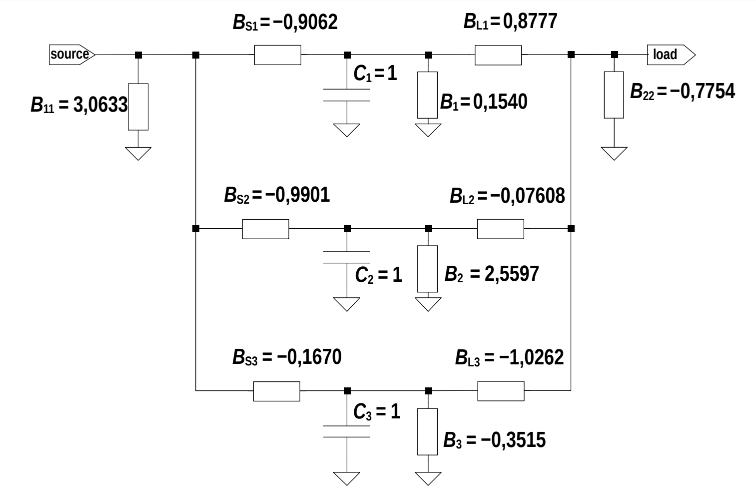
source
load
gS1 = 0,9062
gL1 = −0,8777
gS2 = 0,9901
gL2 = 0,07608
gS3 = 0,1670
gL3 = 1,0262
A B
D C
B11 = 1
C1 = 1
B1 = 0,1255
B22 = −1
C2 = 1
B2 = 1,4935
C3 = 1
B3 = −1,5447

# Żyrator



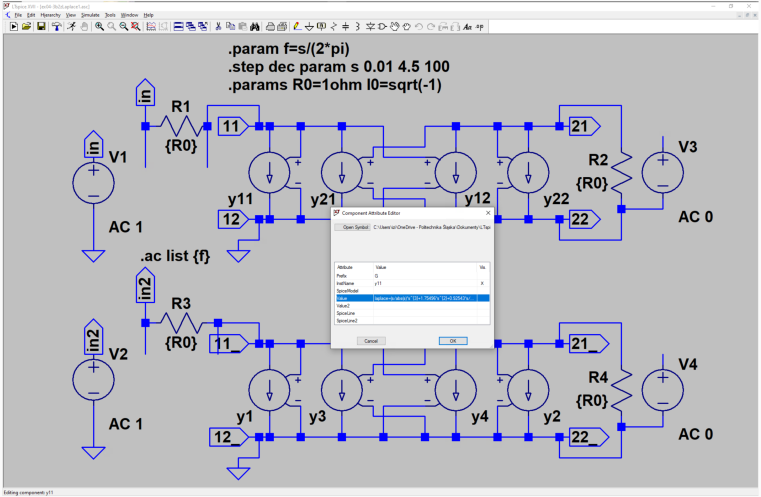

$$\mathrm{Y} = \begin{pmatrix} \mathrm{jg_m} - \mathrm{jg_m} & -\mathrm{jg_m} \\ -\mathrm{jg_m} & \mathrm{jg_m} - \mathrm{jg_m} \end{pmatrix} = \begin{pmatrix} 0 & -\mathrm{jg_m} \\ -\mathrm{jg_m} & 0 \end{pmatrix}.$$

# Przykład 3/5



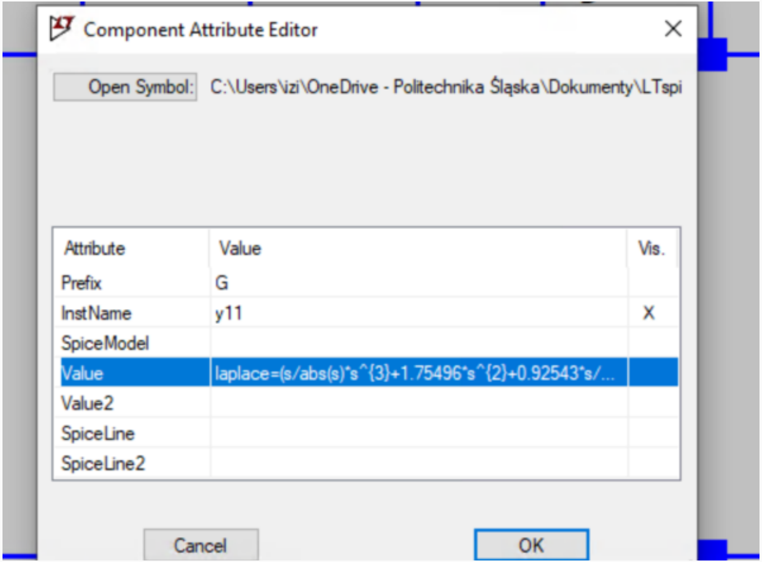
source
load
B_S1 = −0,9062
B_L1 = 0,8777
C_1 = 1
B_11 = 3,0633
B_1 = 0,1540
B_22 = −0,7754
B_S2 = −0,9901
B_L2 = −0,07608
C_2 = 1
B_2 = 2,5597
B_S3 = −0,1670
B_L3 = −1,0262
C_3 = 1
B_3 = −0,3515

# LTSpice 1/3

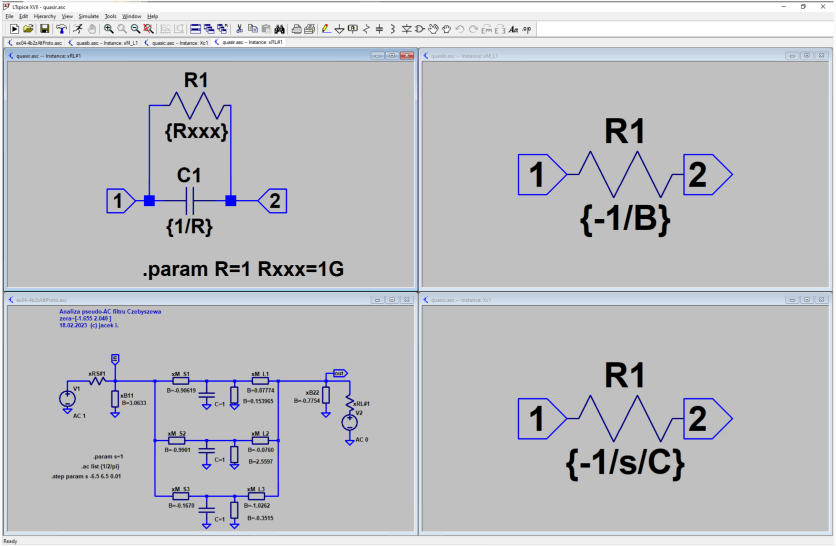


.param f=s/(2*pi)
.step dec param s 0.01 4.5 100
.params R0=1ohm I0=sqrt(-1)
in
R1
{R0}
11
21
V1
AC 1
V3
R2
{R0}
AC 0
y11
y21
y12
y22
12
22
.ac list {f}
in2
R3
{R0}
V2
AC 1
11_
21_
V4
R4
{R0}
AC 0
y1
y3
y4
y2
12_
22_
Component Attribute Editor
Open Symbol:
Attribute
Value
Vis.
Prefix
G
InstName
y11
X
SpiceModel
Value
Value2
SpiceLine
SpiceLine2
Cancel
OK
Editing component: y11

# LTSpice 2/3


Component Attribute Editor
Open Symbol:
C:\Users\izi\OneDrive - Politechnika Śląska\Dokumenty\LTspi
Attribute
Value
Vis.
Prefix
G
InstName
y11
X
SpiceModel
Value
laplace=(s/abs(s)*s^{3}+1.75496*s^{2}+0.92543*s/...
Value2
SpiceLine
SpiceLine2
Cancel
OK

# LTSpice 3/3


LTspice XVII - quasir.asc
quasir.asc -- Instance: xRL#1
R1
{Rxxx}
C1
{1/R}
1
2
.param R=1 Rxxx=1G
quasib.asc -- Instance: xM_L1
R1
1
2
{-1/B}
Analiza pseudo-AC filtru Czebyszewa
zera=[-1.655 2.040 ]
18.02.2023 (c) jacek i.
.param s=1
.ac list {1/2/pi}
.step param s -6.5 6.5 0.01
quasic.asc -- Instance: Xc1
R1
1
2
{-1/s/C}
Ready

# Równania

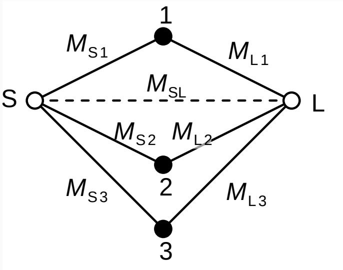
$$Y(s) = sE + j \cdot M \qquad E = \mathrm{diag}(0, 1, \dots, 1, 0)$$

$$M = \begin{bmatrix}
B_{11} & g_{S_1} & g_{S_2} & g_{S_3} & \cdots & g_{S_n} & \cdots & g_{S_{N-1}} & g_{S_N} & g_{SL} \\
g_{S_1} & B_1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & g_{L_1} \\
g_{S_2} & 0 & B_2 & 0 & \cdots & 0 & \cdots & 0 & 0 & g_{L_2} \\
g_{S_3} & 0 & 0 & B_3 & \cdots & 0 & \cdots & 0 & 0 & g_{L_3} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \iddots & \vdots & \vdots & \vdots \\
g_{S_n} & 0 & 0 & 0 & \cdots & B_n & \cdots & 0 & 0 & g_{L_n} \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
g_{S_{N-1}} & 0 & 0 & 0 & \cdots & 0 & \cdots & B_{N-1} & 0 & g_{L_{N-1}} \\
g_{S_N} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & B_N & g_{L_N} \\
g_{SL} & g_{L_1} & g_{L_2} & g_{L_3} & \cdots & g_{L_n} & \cdots & g_{L_{N-1}} & g_{L_N} & B_{22}
\end{bmatrix}$$

# Przykład 4/5

M =

| | S | 1 | 2 | 3 | L | |
|---|---|---|---|---|---|---|
| | 1,00 | 0,91 | 0,99 | 0,17 | 0 | S |
| | 0,91 | 0,13 | 0 | 0 | −0,88 | 1 |
| | 0,99 | 0 | 1,49 | 0 | 0,08 | 2 |
| | 0,17 | 0 | 0 | −1,54 | 1,03 | 3 |
| | 0 | −0,88 | 0,08 | 1,03 | −1,00 | L |



# Obroty

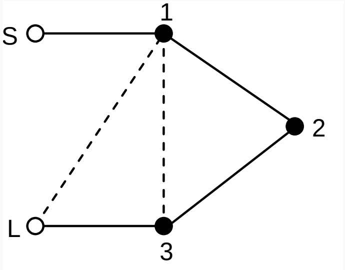
$$Y \mapsto RYR^{-1}.$$

$$R(i,j,\varphi) = \begin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & \cos\varphi & & \cdots & & -\sin\varphi & \cdots \\ & & & & 1 & & & & \\ & & & \vdots & & \ddots & & \vdots & \\ & & & & & & 1 & & \\ & & & \sin\varphi & & \cdots & & \cos\varphi & \cdots \\ & & & \vdots & & & & \vdots & \end{bmatrix} \begin{matrix} \\ \\ \\ i \\ \\ \\ \\ j \\ \\ \end{matrix}$$

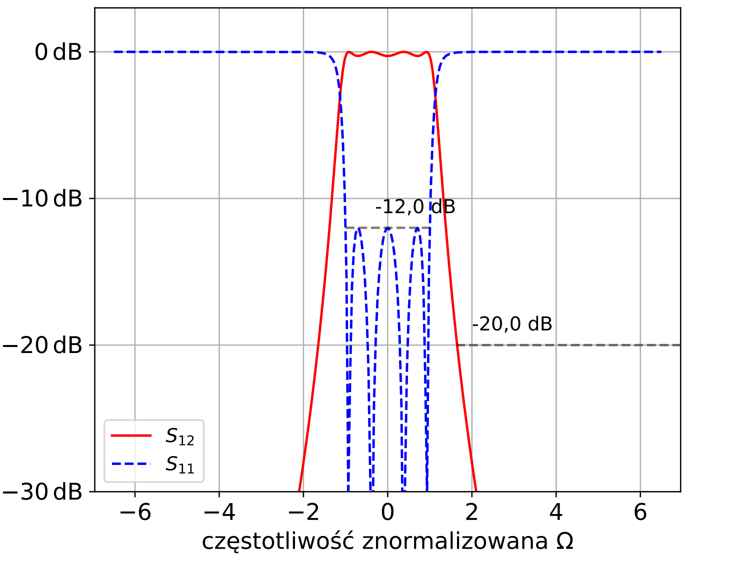
$$\qquad\qquad\qquad i \qquad\qquad\qquad j$$

# Przykład 5/5

$$
\mathrm{M}''' = \begin{pmatrix} 1 & 1{,}35 & 0 & 0 & 0 \\ 1{,}35 & 0{,}83 & 0{,}73 & 0{,}12 & -0{,}41 \\ 0 & 0{,}73 & 0{,}16 & 0{,}99 & 0 \\ 0 & 0{,}12 & 0{,}99 & -0{,}92 & 1{,}29 \\ 0 & -0{,}41 & 0 & 1{,}29 & -1 \end{pmatrix}
$$



# Filtr Czebyszewa 1/4

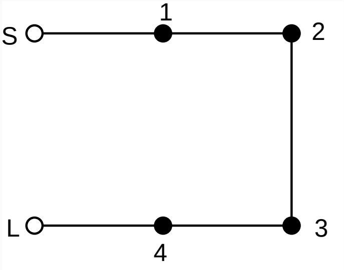


# Filtr Czebyszewa 2/4

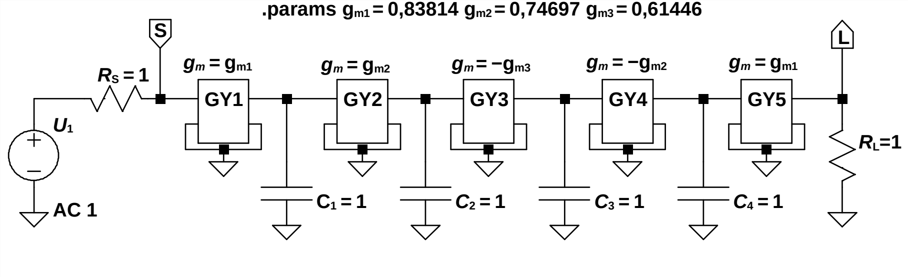
$$M = \begin{pmatrix} 0 & 0{,}492 & 0{,}492 & 0{,}33 & 0{,}33 & 0 \\ 0{,}492 & 0{,}5 & 0 & 0 & 0 & -0{,}492 \\ 0{,}492 & 0 & -0{,}5 & 0 & 0 & 0{,}492 \\ 0{,}33 & 0 & 0 & -1{,}115 & 0 & -0{,}33 \\ 0{,}33 & 0 & 0 & 0 & 1{,}115 & 0{,}33 \\ 0 & -0{,}492 & 0{,}492 & -0{,}33 & 0{,}33 & 0 \end{pmatrix}.$$

# Filtr Czebyszewa 3/4

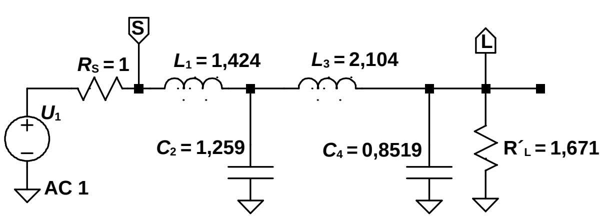
$$M = \begin{pmatrix} 0 & 0{,}838 & 0 & 0 & 0 & 0 \\ 0{,}838 & 0 & 0{,}747 & 0 & 0 & 0 \\ 0 & 0{,}747 & 0 & -0{,}614 & 0 & 0 \\ 0 & 0 & -0{,}614 & 0 & -0{,}747 & 0 \\ 0 & 0 & 0 & -0{,}747 & 0 & 0{,}838 \\ 0 & 0 & 0 & 0 & 0{,}838 & 0 \end{pmatrix}$$



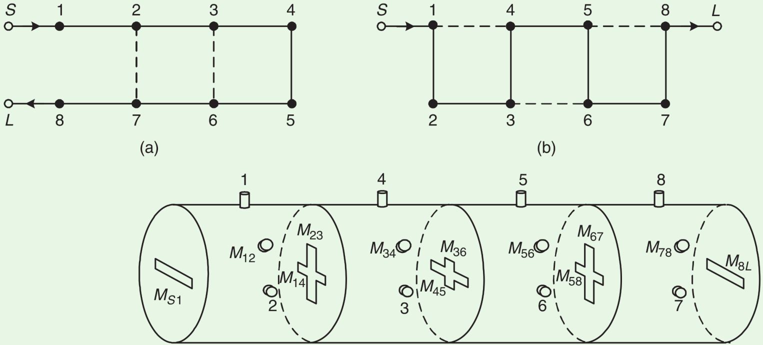
# Filtr Czebyszewa 4/4




a)




b)



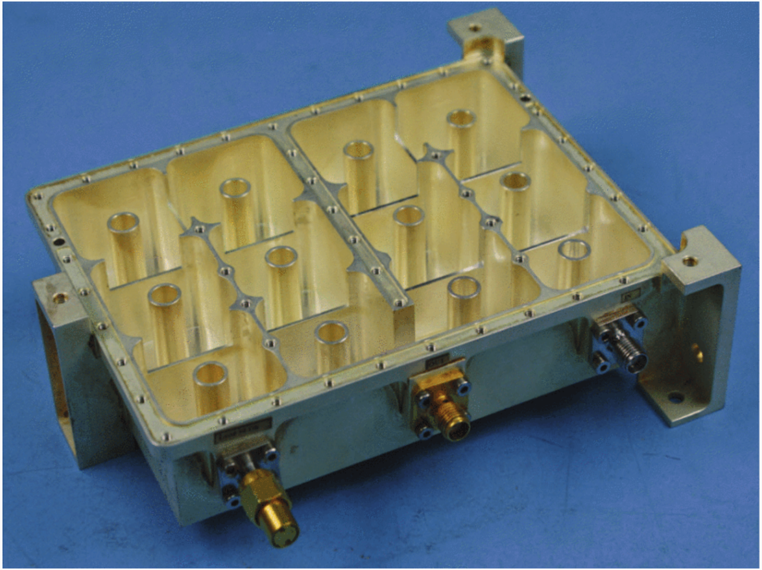

| Physical Cavity No. | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Containing Electrical Resonance Numbers | Vertical Polarization | 1 | 4 | 5 | 8 |
| | Horizontal Polarization | 2 | 3 | 6 | 7 |

(c)

# Tłumienie wtrąceniowe

$$\Delta\alpha = 20\log_{10}\left(\frac{S(0)}{S(0)+\Delta S(0)}\right) \approx \frac{20}{\ln 10}\frac{\beta \cdot T(0)}{Q}.$$

# Rezonatory



# Wnioski

:(