Channel Estimation in MIMO Systems

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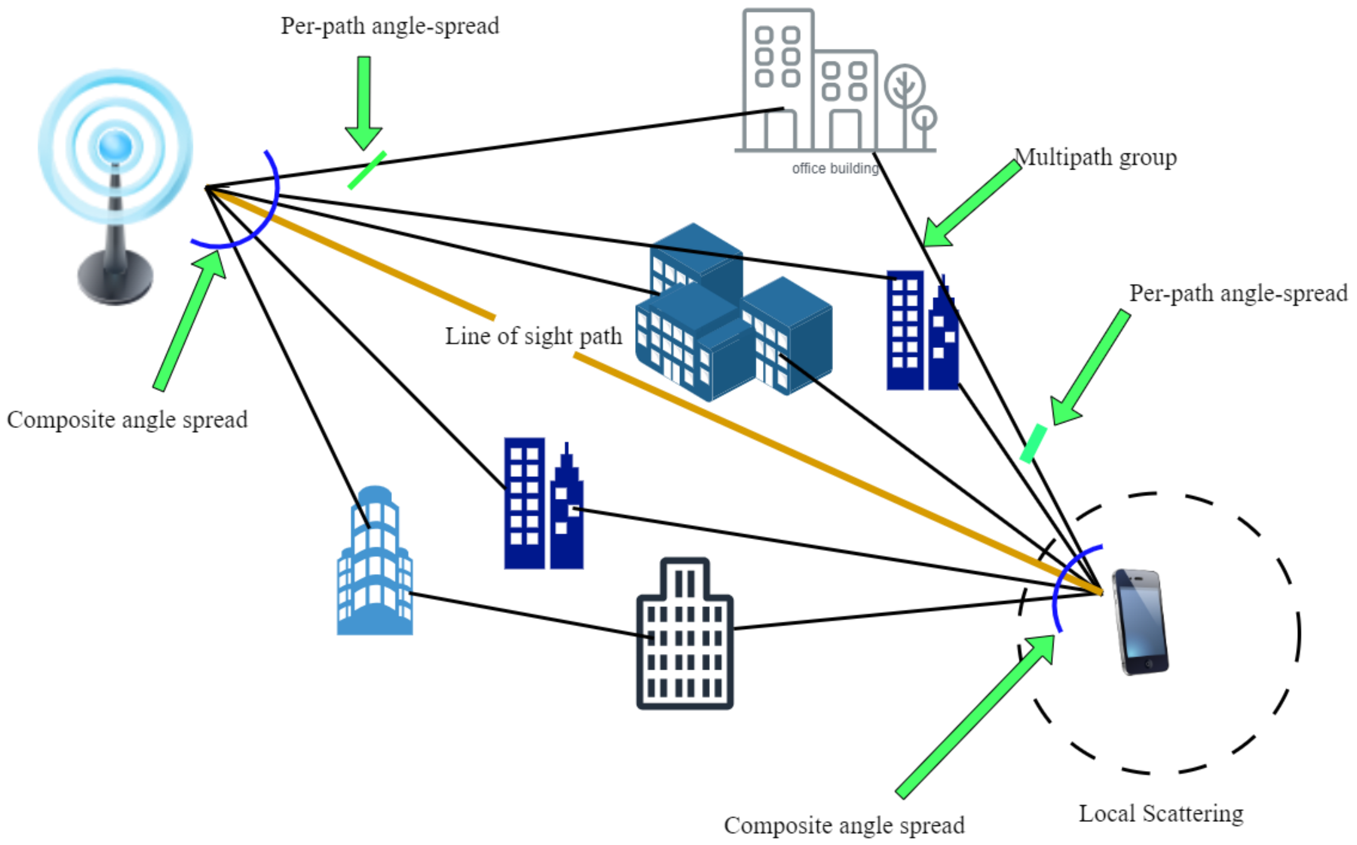
Outline

- Introduction
- OFDM and MIMO System
- Need for estimation for MIMO System
- Channel Estimation Techniques
- Performance comparison
- Challenges
- Solution - Compressive Sensing Channel Estimation
- Conclusion

Wireless Communication Channel



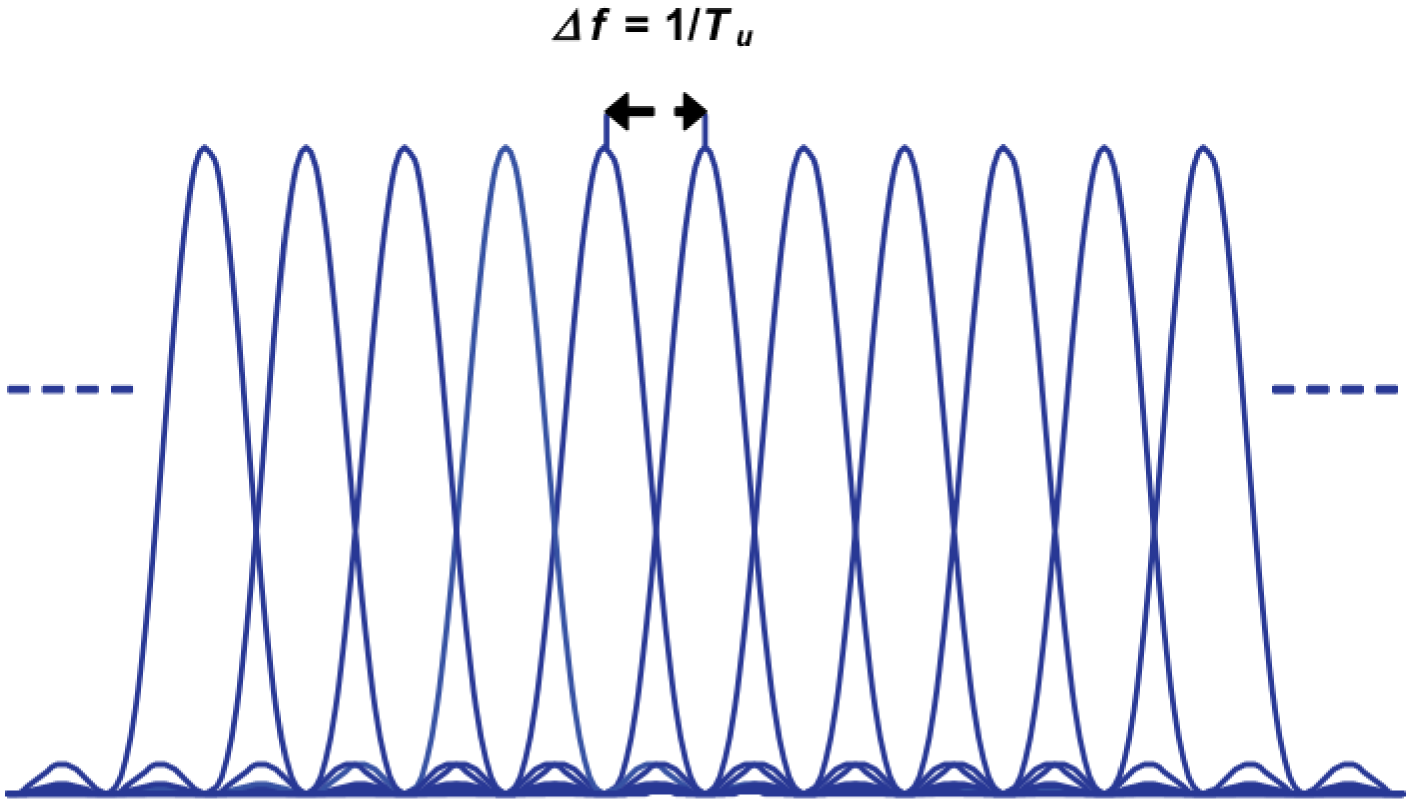
Multicarrier Transmission

- A way to increase overall transmission bandwidth without suffering from increased signal corruption due to radio channel frequency selectivity.
- It converts a frequency selective fading into a flat fading channel making signal detection easier.
- Transmits data over several carrier frequencies simultaneously.

Multicarrier Transmission

- Parallel conversion lowers the data rate and bandwidth in each stream.
- Example is Orthogonal Frequency Multiplexing Access

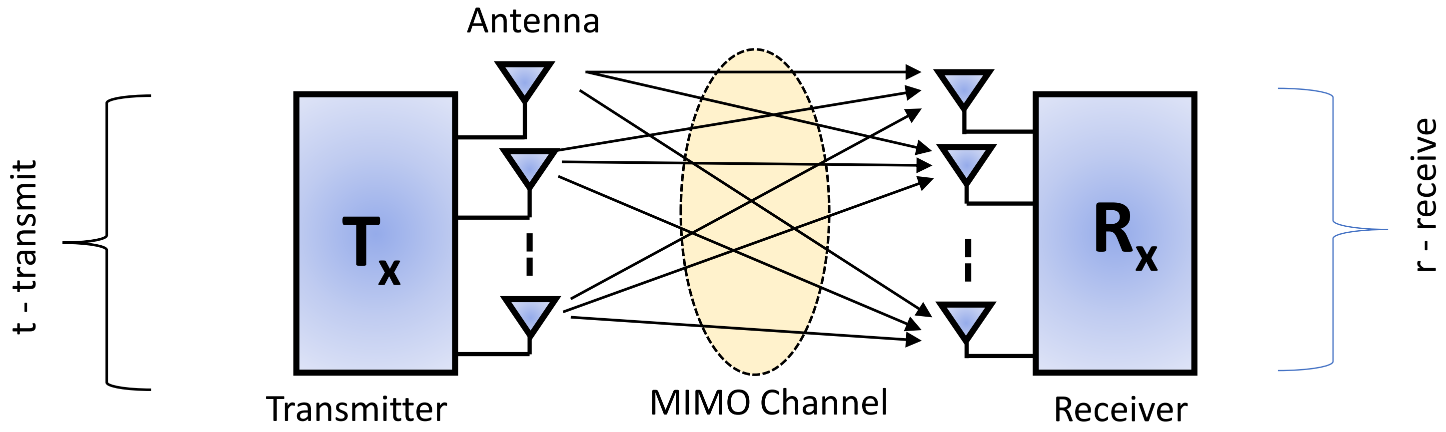
OFDM Overview



OFDM Advantages

- Robustness in multipath environments
- High spectral efficiency
- Simple implementation by FFT
- Low receiver complexity
- Eliminates Inter symbol interference through the use of cyclic prefix
- Less sensitive to sample timing offsets than single carrier systems
- Makes the equalization extremely simple.

Introduction to MIMO System



MIMO is characterized by multiple transmit and receive antennas

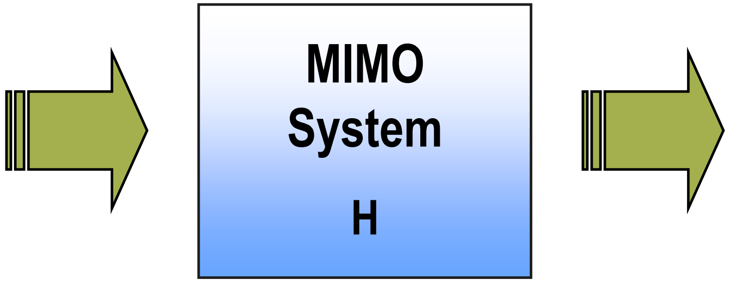
The channel between each T_x – R_x pair is characterized by a complex fading coefficient.

The channel is represented by flat-fading channel Matrix *H.*

MIMO generally improves capacity

MIMO System Model

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}$$



$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}$$

$$\boldsymbol{y}(\boldsymbol{k}) = \boldsymbol{H}\boldsymbol{x}(\boldsymbol{k}) + \boldsymbol{n}(\boldsymbol{k})$$

where ,

$$\mathrm{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ . & & & \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix}$$

is the $r \times t$ complex channel matrix

- Estimating H is the problem of **‘Channel Estimation’**

MIMO Capacity Formula

- Capacity is given in terms of the mutual information between channel input vector X and output vector Y.
- Mutual information for multiantenna:

$$I(X\!:\!Y) = B\log_2 \det[I_{mr} + HR_X H^H]$$

- MIMO capacity is achieved by increasing mutual information:

$$C = \max_{p(x)} B\log_2 \det[I_{mr} + HR_x H^H]$$

Need for channel estimation

Chanel State Information (CSI) is critical in MIMO systems.

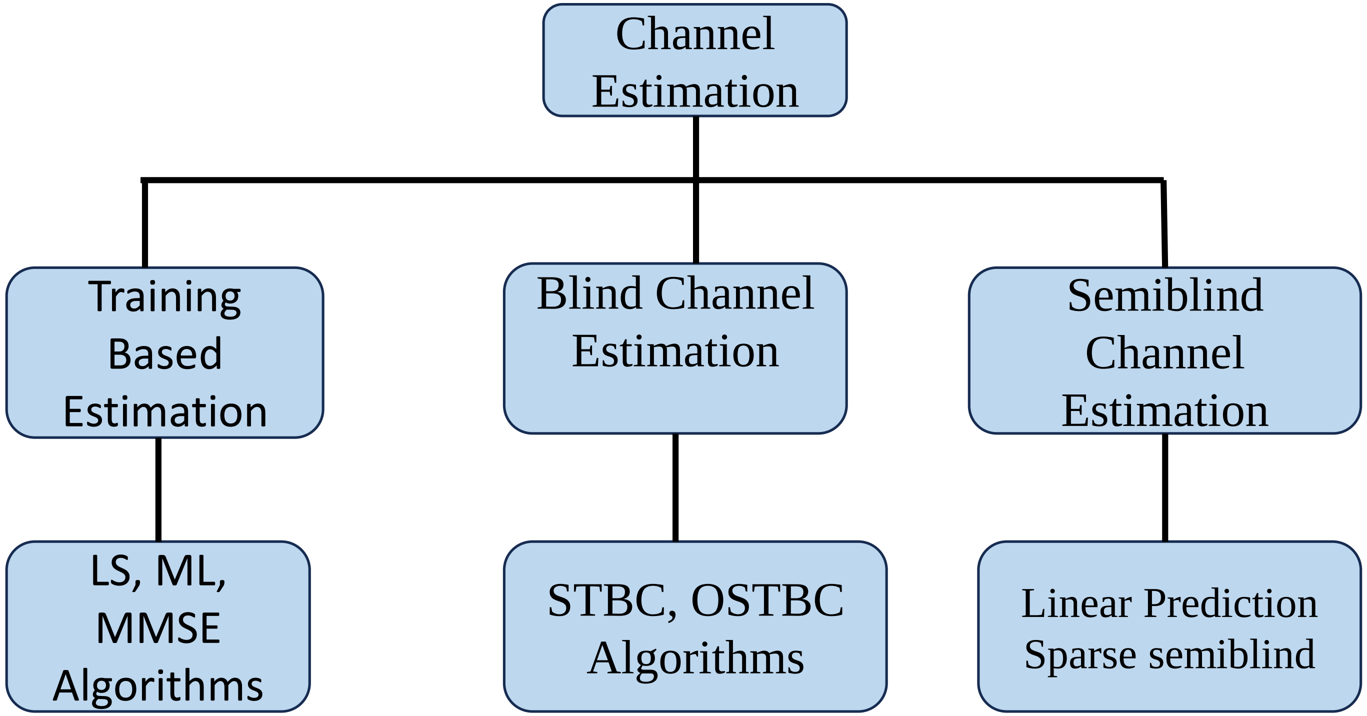
Channel estimation is key to MIMO gains.

As the diversity of the MIMO system increases, the operating SNR increases- calling for more robust estimation strategies.

- Ensure delivery of service.
- Determining the optimal communication path
- Enhance quality of service
- Improve performance through rate adaptation.

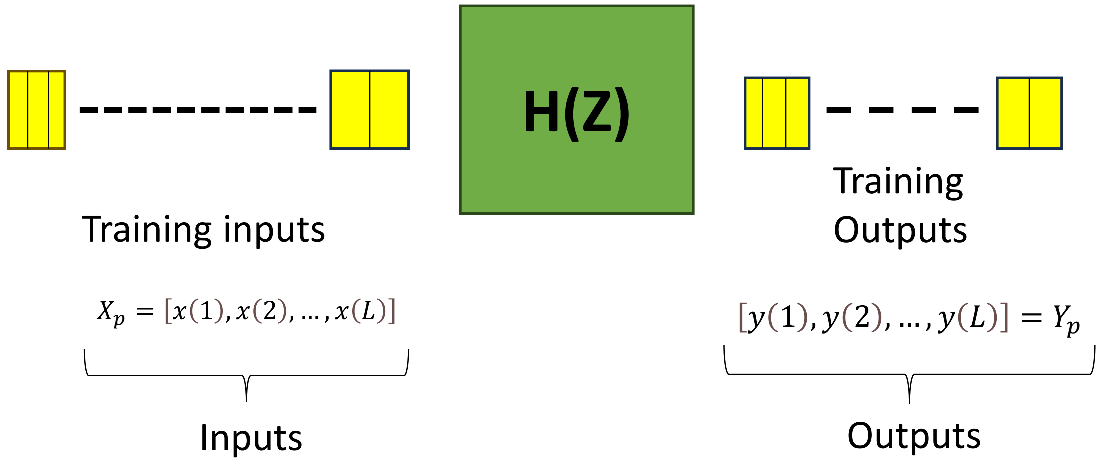
Channel Estimation Techniques

- The techniques can broadly be classified as:
  - ✓ Pilot Assisted Channel Estimation
  - ✓ Blind Channel Estimation
  - ✓ Semi-blind channel Estimation



Training-based Channel Estimation

A model for training symbol-based channel estimation.



- To formulate the least squares cost function, $min\|Y_P - HX_P\|_F^2 \cdot$
- The estimate of H is given as $\widehat{H}_{LS} = Y_P X_p^+ \cdot$
- The mean square error of this LS channel estimate is given as:
- $MSE_{LS} = E\left\{\left(H - \widehat{H}_{LS}\right)^H (H - H_{LS})\right\} = E\{(H - X^{-1}Y)^H (H - X^{-1}Y)\} = E\{z^H (XX^H)^{-1} z\} = \frac{\sigma_z^2}{\sigma_x^2} \propto \frac{1}{SNR}$

Pros and Cons

Pros

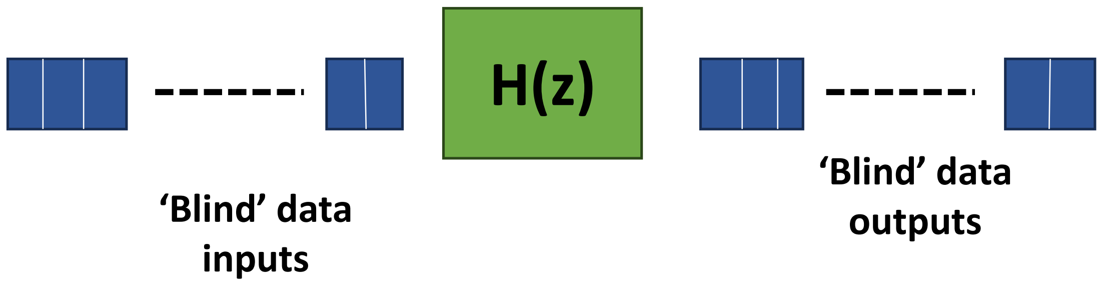
- Lower complexity simplifying receiver design.
- Established approach- widely studied.
- Easy implementation.

Cons

- Bandwidth inefficiency
- Limited adaptability
- High estimation errors

Blind Channel Estimation

Model for Blind Channel Estimation



Blind Channel Estimation

- No pilot training necessary.
- Statistics that give the autocorrelation matrix R_y are:

  - Source covariance is known, $E(x(k)x(k)^H) = \sigma_s^2 I_t$

  - Noise covariance is known, $E(v(k)v(k)^H) = \sigma_n^2 I_r$
- The channel estimation by least square optimization procedure is

$$\hat{h} = \arg \min_{\|h\|=1} h^H Q Q^H h$$

Blind Channel Estimation

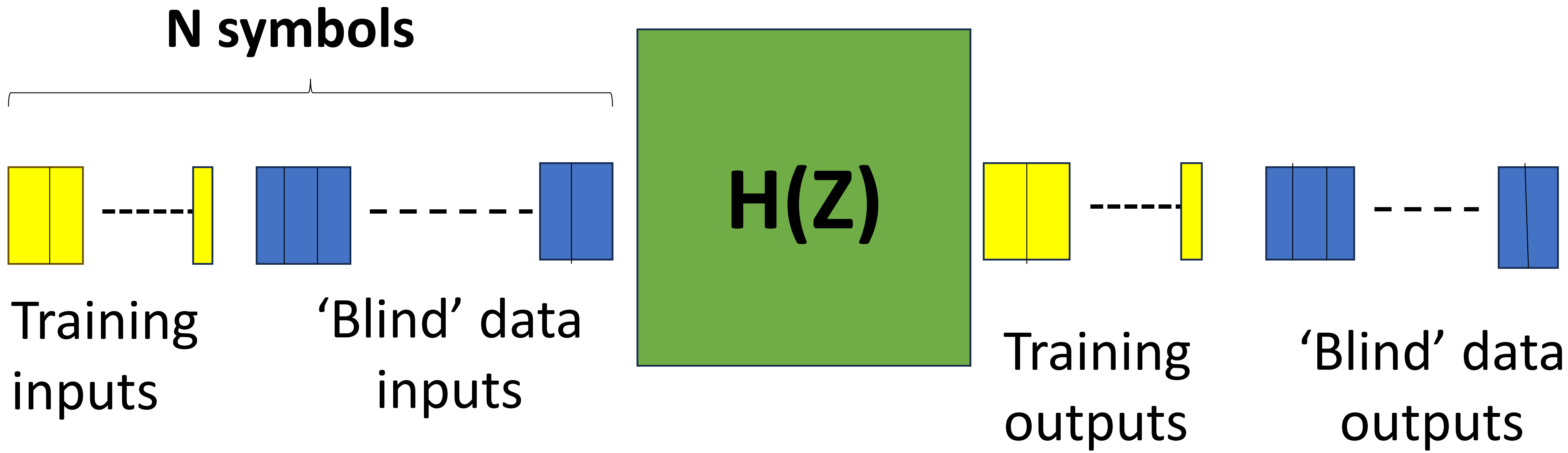
Pros

- Statistical approach
- Best approach to mitigate pilot contamination.
- Efficient spectrum use.

Cons

- Higher computational complexity.

Semi-blind Channel Estimation



Semi-blind Channel Estimation

- A hybrid approach that utilizes limited pilot symbols for improving the convergence speed and helps to track time-varying channels.
- The estimated channel coefficients evaluated are fed back to the transmitter.
- The cost function is estimated as: $\min_{\hat{h}_v} \left\{ \left\| Y - X\hat{h} \right\|^2 + \alpha \left\| \bar{B}\hat{h} \right\|^2 \right\}$

Semi-blind Channel Estimation

Pros

- Higher bandwidth efficiency.
- Channel behavior can be tracked efficiently.

Cons

- Poor performance in Rayleigh fading scenarios.
- Complexity trade-offs.
- Statistical assumptions.

Other Techniques

- Other techniques include:

I. DFT Based Channel Estimation
II. Direction-Directed Channel Estimation
III. Channel Estimation using superimposed Signal
IV. EM (Expectation-Maximization) algorithm- Based Channel Estimation
V. Among many more others

Performance Metrics

- Techniques include:
  - ❑MSE
  - ❑NMSE
  - ❑SINR
  - ❑BER
  - ❑Cramer-Rao Bound (CRB)
  - ❑Among others

Mean Square Error (MSE)

- Widely used metric to evaluate the accuracy of channel estimation methods.
- A measure of the average squared difference between the actual channel and the estimated channel.
- Interpretation: Lower MSE indicates a more accurate and precise channel estimate.

Mean Square Error (MSE)

- Mathematical expression:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left\| h_i - \widehat{h}_i \right\|_2^2$$

Where:

- N – total number of channel coefficients.
- $\|\cdot\|_2$ denotes the Euclidean norm

Normalized Mean Square Error (NMSE)

- MSE divided by the variance of the actual channel.
- Normalized to account for varying signal power levels in different channels.
- A lower NMSE indicates better channel estimation performance.

Normalized Mean Square Error (NMSE)

- Mathematical Expression

$$NMSE = \frac{MSE}{\|h\|_2^2}$$

Where:

- h – represents the true channel vector.
- $\|\cdot\|_2$ denotes the Euclidean norm.

Signal-to-Interference Noise Ratio (SINR)

- Measure of the signal power relative to the interference and noise power.

- An accurate channel estimate is essential for maximizing the SINR and achieving reliable communication.

- Higher SNR indicates a stronger signal relative to noise.

Signal-to-Interference Noise Ratio (SINR)

- Mathematical expression:
  - In logarithmic scale:

$$SNR(\mathrm{d}B) = 10 \cdot \log_{10}\left(\frac{Signal\ Power}{Noise\ Power}\right)$$

  - In linear scale:

$$SNR = \frac{Signal\ Power}{Noise\ power}$$

Cramer-Rao Bound (CRB)

- Provides a theoretical lower bound on the variance of any unbiased channel estimator.
- Performance measured by its covariance given as

$$C_\theta = E\left[\left(\hat{\theta} - \theta\right)\left(\hat{\theta} - \theta\right)^H\right]$$

- Gives a lower bound on the achievable estimation error.

- The CRB on the covariance of an un-biased estimator is given as

$$C_\theta \geq J^{-1}$$

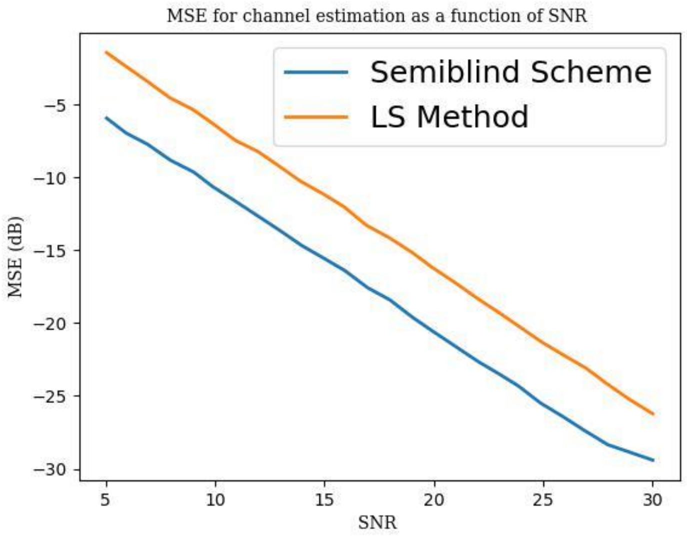
where

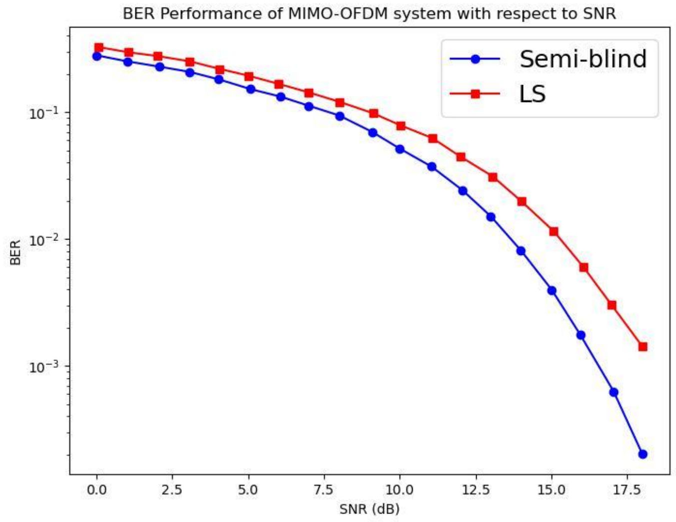
$$J = E\left\{\left[\frac{\partial \ln p(\overline{\omega},\theta)}{\partial\theta}\right]^T \left[\frac{\partial \ln p(\overline{\omega},\theta)}{\partial\theta}\right]\right\}$$

Computational Complexity

Algorithm	Complexity
Blind Channel Estimation	$O(N^3)$
Semi-Blind Channel Estimation	$O(\min\{MN^2, M^2N\})$
Pilot Assisted Channel Estimation	$O(N_P N_s)$

Where M and N are the number of antennas at the base station and User equipment respectively for semi-blind channel estimation.
N_P - number of pilots.
N_s - number of subcarriers.





Simulation

- A MIMO-OFDM system with 2 transmit and 4 receive antennas with QPSK modulation.
- Rayleigh channel model is used.
- Estimation performance is measured in terms of MSE of the estimate of the channel given by: $MSE = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} \left\| \hat{h}_n - h_n \right\|^2$

Challenges in Channel Estimation for 5G and 6G Systems

- Reduce redundancy in the acquisition of CSI at the Base Station.
- Efficient techniques with low computational techniques.
- Practical channel estimation techniques that will cope with time-varying conditions.
- Techniques to exploit channel sparsity in MIMO.

Solution – Compressive Sensing

- Why ?
  - Exploiting sparsity.
  - Reducing complexities.
  - Fast reconstruction
  - Adapting to varying conditions
  - Improving channel estimation accuracy

Project Goals

- Compare different compressed sensing strategies (e.g., OMP, AMP, CoSaMP) and legacy channel estimation techniques in MIMO systems.
- Formulate and design a novel wireless channel estimation algorithm based on compressed sensing (CoSaMP).
- Performance evaluation of designed algorithms and comparison with existing ones.

What is Compressive Sensing (CS)?

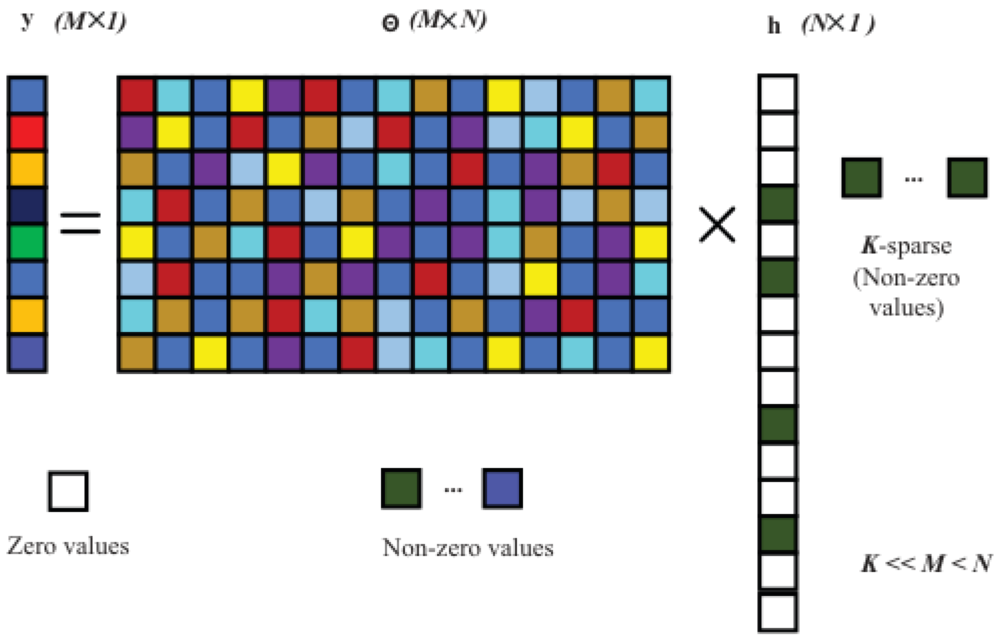
- CS theory asserts signals and images can be recovered from fewer samples or measurements than the traditional paradigm.
- Number of samples needed for exact recovery depends on particular reconstruction algorithm.
- It relies on two principles:

  - Sparsity – Is the signal vector to be recovered sparse?

  - Incoherence- the sparse signal can’t be sparse in the domain in which it is sampled.

Fundamental of Compressive Sensing

- Representation of Measurements in CS considering a noiseless scenario



Fundamentals of CS

- A noisy scenario defined by:

$$y = \Theta h + z$$

Where Θ is the measurement matrix, h is the signal said to be sparse and to be recovered and z is the noise vector.

- h can be recovered by y if $K > M$ and $M > N$.

Main tasks in CS

1

Designing a good measurement matrix.

- Matrices with large compression effects and robustness against modeling errors are desired.
- It should satisfy Restricted Isometric Property (RIP)

2

Designing a good signal recovery algorithm

- Fast and robust algorithms are desired.

Restricted Isometric Property

- A sufficient condition for a stable solution for both K-sparse and compressible signals that the sensing matrix must satisfy for any arbitrary k-sparse vector.
- For some $0 < \delta < 1$, the measurement matrix should satisfy:

$$1 - \delta \leq \frac{\|\Theta h\|_2^2}{\|h\|_2^2} \leq 1 + \delta$$

Where δ is the Restricted Isometric Constant

Compressive Sampling Matching Pursuit

- An iterative method used to reconstruct the vector signal from compressive samples.

- Provides comparable performance to that of best optimization-based approaches with low computational complexity.

- Has tighter bounds on its convergence and performance.

- It has a complexity of :

$$O(mn)$$

Algorithm Steps

- Identification
- Support Merge
- Estimation
- Pruning
- Sample Update

Input:

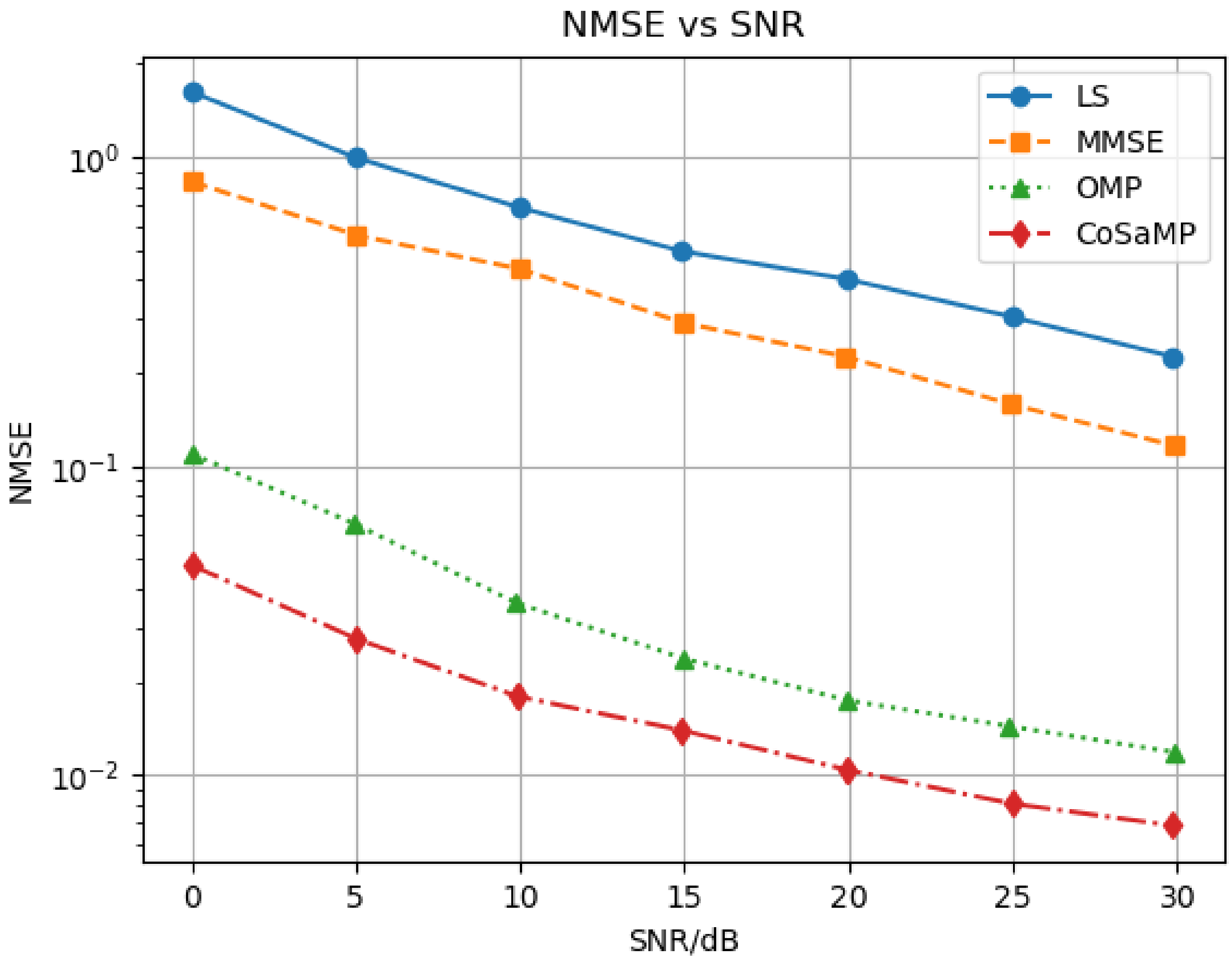
- Transform matrix $\mathbf{\Psi}$, Measurement matrix $\mathbf{\Phi}$
- CS matrix $\mathbf{A}$: $\mathbf{A} = \mathbf{\Phi}\mathbf{\Psi}$
- Measurement vector $\mathbf{y}$
- K being the signal sparsity
- Halting criterion

Output:

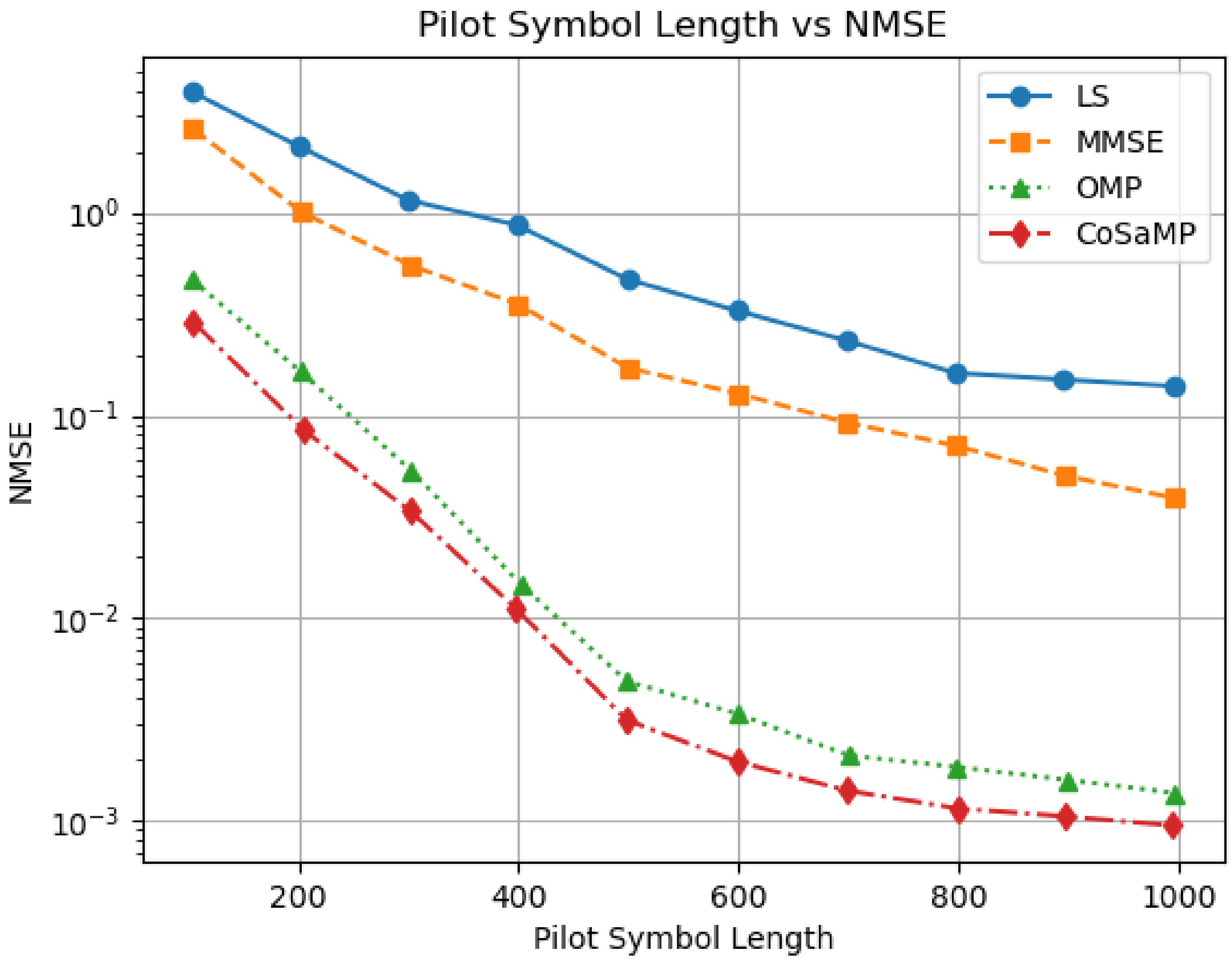
- K-sparse approximation $\hat{\mathbf{x}}$ of the target signal

(1) $\mathbf{x}_0 \leftarrow 0$, $\mathbf{r} \leftarrow \mathbf{y}$, $i \leftarrow 0$
(2) **while** halting condition false **do**
(3) $i \leftarrow i + 1$
(4) $\mathbf{z} \leftarrow \mathbf{A}^*\mathbf{r}$ *(signal proxy)*
(5) $\Omega \leftarrow \mathrm{supp}(\mathbf{z}^{2K})$ *(support of best 2K-sparse approximation)*
(6) $T \leftarrow \Omega \cup \mathrm{supp}(\mathbf{x}_{i-1})$ *(merge supports)*
(7) $\tilde{\mathbf{x}} = \arg\min_{\tilde{\mathbf{x}}:\mathrm{supp}(\tilde{\mathbf{x}})=T} \|\mathbf{A}\tilde{\mathbf{x}} - \mathbf{y}\|_2^2$ *(solve least squares)*
(8) $\mathbf{x}_i \leftarrow \tilde{\mathbf{x}}^K$ *(Prune: best K-sparse approximation)*
(9) $\mathbf{r} \leftarrow \mathbf{y} - \mathbf{A}\mathbf{x}_i$ *(update current sample)*
(10) **end while**
(11) $\hat{\mathbf{x}} \leftarrow \mathbf{x}_i$
(12) **return** $\hat{\mathbf{x}}$

Performance Results [6]



Performance results



Challenges

- Require a priori knowledge of the sparsity level while practical environment, channel sparsity is unknown.

Conclusion

- Why Channel estimation?
- Techniques of channel estimation
- Performance of different techniques
- Challenges in channel estimation
- Proposed Solution

Questions

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