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PHD THESIS ABSTRACT
Random simulations of geometric objects on the plane and their using in the theory of representative volume element

PhD thesis under supervision of Prof. Dr. Władimir Mitiuszew

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1 Root of the research

The subject of this thesis is to expand applications of basic sums to randomly generated geometric objects on the plane. Research conducted recently shows that basic sums can play significant role in many aspects, e.g. composites modeling. Moreover, the development of the new composite volume element [7] has made that geometric information has appeared in basic sums. Due to creation of efficient and fast algorithms [9, 10], which describe basic sums values and structures, their usage became much more convenient and efficient. However, the whole system of basic sums and their usage is connected with the analysis containing non-overlapping circles [8]. We encountered a problem of basic sums usage while treating as geometric information carrier, in order to identify geometric features of images including random geometric objects on the plane, which are invisible to the human, but significant [3–5, 12]. In this thesis I present algorithms of generating random distribution of geometric objects and their analysis by the use of basic sums. The important element of this thesis is to construct to conformal mappings of some multiply connected domains. Together with basic sums they create a systematic tool not only in image processing but also in the determination of effective composite features [6]. The last part of this thesis is the description of basic sums usage in e.g. computational biology, which have become possible thanks to procedures described earlier in this work.

This thesis describes some algorithms, methods and research results which was created based on scientific cooperation with Prof. Władimir Mitiuszew. Most results presented in this thesis was published in author’s publications and in joint papers. Mentioned works were numbered in the bibliography section (positions [RCZ1, RCZ2, RCZ3, RCZ4, RCZ5, RCZ6, RCZ7, RCZ8]).

2 Thesis aim

The main thesis aims are as follows:

1. To describe methods of using basic sums to separate geometric features from an image.

2. To describe algorithms to generate distribution of some geometric objects and characteristics of these distributions by the use of basic sums.

3. To prove effective construction of conformal mapping for some multiply connected domains.
4. To apply basic sums to formulate criteria of collective behavior of bacteria as well as other usage.

3 Actual knowledge status

In theory of image recognition the dominant attitude is to extract visible features from the analyzed image. The next step in image analysis can be the process of extracting some invisible features we do not realize they exist. We can reduce this issue to the construction of set $G$ containing some parameters describing geometry which we can consider as some representation of this image. Based on current knowledge we can distinguish two approaches to the set description. First description defines set $G$ as the set of all $n$-point correlation functions. Such definition of $G$ set is correct, because $n$-point correlation functions entirely describe geometry of considered image. Such representation is widely used in micro structures analysis and their effective features. Unfortunately, practical use of $n$-point correlation functions is strongly limited because of computational difficulties. This thesis focuses on alternative approach to set $G$. This representation is obtained from the formula of effective properties of composite. The set $G$ is introduced as follows:

$$G = \{e_p, \ p \in \mathcal{M}_e\},$$

(3.1)

where $e_p$ is basic sums or $e$-sums, and $\mathcal{M}_e$ is some multi index set. Using this representation is justified for images which include non-overlapping circles. Theory that basic sums can be used in image processing was formulated for the first time in PhD thesis [11]. One of the main goals of this work is to use set $G$ in form (3.1) in order to characterize images containing non-circular geometric objects. Apart from using basic sums as geometric information carrier, we also need to distinguish:

- the determination of effective properties of composite;
- theory of representative volume element (RVE);
- the stir process in composite materials theory;
- the problem of optimal packing of disks on the plane.

4 Scientific authenticity of this thesis

This thesis has come up with new outcomes as follows:
• two methods of using basic sums as geometric information carrier were described in case analysing distribution of non-circular geometric objects - approximation method and conformal mapping method;

• algorithms generating random distributions of non-overlapping geometric objects (line segments, stadiums) were presented and created distributions with basic sums usage were characterized;

• conformal mapping of complex plane with non–overlapping disks onto the plane with non–overlapping slits (line segments) of given inclinations in iterative form was constructed and analysed;

• conformal mapping of square with circular holes onto the square with non–overlapping slits (line segments) of given inclinations was constructed;

• basic sums were used in order to formulate new quantitative criterion of collective behavior of bacteria;

• algorithm implementation was described which gives the reason to have a hypothesis that due to comparative analysis with basic sums usage we can create simulative models of real locations of bacteria.

5 Research methodology

This thesis used:

• symbolic computations;

• numerical computations;

• computer simulations;

• comparative analysis;

• stochastic methods (Monte Carlo method).

6 Practical and theoretical meaning of this thesis

• Presented methods of approximation and conformal mapping yield constructive implementation of basic sums as geometric information carrier order to analyze random distributions of geometric objects.

• Comparative analysis of random distributions of geometric objects which is generated by various algorithms was conducted.
• Presented conformal mappings allow to wide possibilities to use basic sums order to analyze distributions of geometric objects.

• Basic sums were used in a new area of science, i.e. computational biology - the new criterion of collective behavior of bacteria and real distribution of bacteria simulation.

7 Overview and the most important thesis results

Chapter 1 includes introduction to the thesis subject by describing and stating issues. Current knowledge status was also included here along with formal definitions of analyzed issues and description of some basic sums usage methods. Important part of this chapter is introducing elements of methodology used in this thesis. Firstly, basic sums were presented as geometric information carrier and some tool which can be used in analyzing images including non-overlapping circles. Secondly, two methods which let use basic sums as geometric information carrier in case analyzing distribution of non-circular objects were described in detail. The first method relies on geometric object approximation thanks to circles, the second one is based on the definition of conformal mapping. It is important to mention that this chapter also includes the comparison of numerical methods with symbolic computations in order to present current model of scientific calculation.

Chapter 2 briefly presents current scientific results regarding generation of non-overlapping circles distribution with the use of basic sums. This chapter includes two crucial elements: the first element is the description of algorithms which generate some random distributions of geometric objects. Some examples of presented algorithms:

• generating random distributions of line segments with separation parameter via RSA method;
• generating random distributions of line segments with separation parameter based on random walks;
• generating random distributions of stadiums with some parameters via RSA method;
• generating random distributions of stadiums with directions of locations stated deterministically.
Second important element included in this chapter is the characterization of received random distribution of geometric objects with the use of $e$-sum. Therefore, basic sums are used which correspond to functional of effective features accurate to within $O(\nu^5)$. Basic sums values $e_2$, $e_{2.2}$, $e_{3.3}$, $e_{4.4}$, $e_{2.2.2}$, $e_{3.3.2}$ i $e_{2.2.2.2}$ are estimated via Monte Carlo method. Received values are fundamental parameters characterizing geometric objects distribution, which are generated by mentioned algorithms.

Chapter 3 describes conformal mapping constructions between multiply domain with non-crossing line segments versus canonical circular domain. In the first part of this chapter effective way to construct conformal mapping was presented, which transforms complex plane with non-overlapping circles onto plane with non-crossing line segments. Conformal mapping as the iterative formula was received, which was analyzed via numerical analysis. The second part of this chapter describes the construction of conformal mapping which transforms a square including non-overlapping circles onto a square including non-crossing line segments. Presented model, and the discussion made over the possibility to build the inverse function, have fundamental meaning for the current research as they might allow to use $e$-sums with the use of conformal mapping method in order to analyze distribution of non-crossing line segments. The analysis of line segments distribution (or curves fragments) has crucial meaning in practical use, e.g. fracture mechanics.

The main topic of chapter 4 is the introduction of practical example of basic sums use. New quantitative criterion of bacteria behavior was presented. Real bacteria distribution was considered, in this case we had 31 images with bacteria the kind of bacillus subtilis. Using algorithms of analysis and image processing we read basic data concerning bacteria geometry, which were structured as non-overlapping line segments (first case) as well as non-overlapping stadiums (second case). Then, for comparative purposes, simulation of random line segments (and stadiums - depending on a model) were performed with the use of algorithms presented in the chapter 2, which we consider as an example of chaotic distribution of bacteria. Proper basic sums values were calculated for such theoretical distributions and for real bacteria distribution, basic sums calculations characterize these distributions. Then, received basic sums values sets were compared with each other. Qualitative comparative analysis proved a significant difference between two $e$-sums sets (in case of bacteria shape modelling by both line segments and stadiums). It lead to the assumption of
collective bacteria behavior. In the chapter we also proved it is possible that using an algorithm to generate random stadiums distribution with deterministically-stated directions of locations it is possible to implement an algorithm which would simulate real bacteria distribution. The last element of this chapter is to point the possibility to use e-sums as parameters describing organizing molecules in liquid crystals versus some currently used limitations of order parameters.

The thesis is ended by Appendixes where the elements of complex analysis and the Eisenstein sums are outlined.

References

Author’s publications


**Other publications**


